Macroeconomic costs of gender gaps in a model with entrepreneurship and household production*

David Cuberes  
Clark University

Marc Teignier  
Universitat de Barcelona and BEAT

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Abstract

This paper examines the quantitative effects of gender gaps in entrepreneurship and workforce participation in an occupational choice model with a household sector and endogenous female labor supply. Gender gaps in workforce participation have a direct negative effect on market, while gender gaps in entrepreneurship affect negatively market output not only by reducing wages and labor force participation but also by reducing the average talent of entrepreneurs and aggregate productivity. We estimate the effects of these gender gaps for 37 European countries, as well as the United States, and find that gender gaps cause an average loss of 17.5% in market output and 13.2% in total output, which also includes household output. Interestingly, the total output loss would be similar (12%) in a model without household sector, since the market output loss is larger when the female labor supply is endogenous. Eastern Europe is the region with lowest income fall due to gender gaps, while Southern Europe is the region with the largest fall. Northern Europe is the region with the largest productivity fall, which is due to the presence of high gender gaps in entrepreneurship.

JEL classification numbers: E2, J21, J24, O40.

Keywords: gender inequality, household production, factor misallocation, aggregate productivity.

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1 Introduction

Gender inequality is present in many socioeconomic indicators around the world in both developed and developing countries. In this paper, we study two important aspects of gender inequality in the labor market, namely the low female participation in the labor market and the low presence of women in entrepreneurial activities. Worldwide, women are underrepresented in the labor market, but this is especially the case in entrepreneurial occupations like employers and self-employed workers. Our focus is on Europe, where the female-to-male ratio is between 0.76 and 0.94 in terms of labor force participation in the four regions considered, between 0.45 and 0.64 for self-employed workers, and between 0.31 and 0.43 for employers, as we can see in Table 1.

Table 1: Gender differences in the labor market, by European region

<table>
<thead>
<tr>
<th>(female to male ratios)</th>
<th>Labor force part.</th>
<th>Self Employed</th>
<th>Employers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eastern Europe</td>
<td>0.88</td>
<td>0.61</td>
<td>0.43</td>
</tr>
<tr>
<td>Northern Europe</td>
<td>0.94</td>
<td>0.52</td>
<td>0.34</td>
</tr>
<tr>
<td>Southern Europe</td>
<td>0.76</td>
<td>0.45</td>
<td>0.32</td>
</tr>
<tr>
<td>Western Europe</td>
<td>0.84</td>
<td>0.64</td>
<td>0.31</td>
</tr>
<tr>
<td>United States</td>
<td>0.86</td>
<td>0.71</td>
<td>0.41</td>
</tr>
</tbody>
</table>

The main goal of this article is to quantify the aggregate effects of the aforementioned gender gaps on aggregate productivity and income in Europe taking into account the presence of the household sector. To do that we extend the general equilibrium occupational choice model in Cuberes and Teignier (2016) to incorporate the household sector. In the model, agents are endowed with a random skill level, based on which they decide to work as either employers, self-employed workers, market workers, or, in the case of women, household sector workers. An employer in this model produces the consumption good using a span-of-control technology that combines his or her managerial skills with capital and workers. On the other hand, a self-employed worker can produce the same consumption.

1See the World Development Report 2012 (World Bank, 2012) for a comprehensive review.
good using a similar technology - adjusted by a productivity parameter - but without hiring any workers. Finally, women also have the possibility of producing the economy’s unique consumption good at home. In the model, women are identical to men in terms of their managerial skills but they are subject to several exogenous constraints in their labor market choices. As we show below, these restrictions distort the occupational allocation and reduce aggregate productivity and income per capita.

We calibrate the model to match the United States data on employment status and household production and estimate the gender gaps for 37 European countries, as well as the United States. Our numerical results show that the gender gaps considered here cause an average output loss of 17.5% when only considering market output and 13.2% when including also household output. Interestingly, in a model without household sector the estimated output loss would be somewhat lower, 12%, even if the increase in household production is not taken into account. The reason for this is that the market output loss is larger when the household sector is taken into account, since some women choose to leave the labor force to produce in the household sector when the gender gaps are introduced and the market conditions get worse. When looking at the results by region, we see that Eastern Europe is the region with lowest income fall due to gender gaps, while Southern Europe is the region with the largest fall. Northern Europe, on the other hand, is the region with the largest productivity fall which is due to the presence of high gender gaps in entrepreneurship.

Several articles in the literature study the relationship between gender inequality and economic performance, like Galor and Weil (1996), Lagerlöf (2003), Greenwood et al. (2005), Doepke and Tertilt (2009), Esteve-Volart (2009), Fernandez (2009), or Ngai and Petrongolo (2017). To our knowledge, there are very few papers that quantify the macroeconomic effects of these gender gaps in the labor market. The International Labor Organization provides some estimates of the output costs associated with labor gender gaps in the Middle

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2 We could certainly allow men to work in the household sector too. However, if we make the plausible assumption that women are more productive at home than men, the main results of the paper would still hold.

3 See Cuberes and Teignier (2014) for a critical literature review of the two-directional link between gender inequality and economic growth.

There is also a large literature that incorporates a household sector in macroeconomic models to understand its importance for the aggregate economy. See, among others, Prescott (2004), Gollin, Parente, and Rogerson (2004), Greenwood et al. (2005), Rogerson (2007, 2008), Ngai and Pissarides (2011), Guner et al. (2012a, 2012b), Moro et al. (2017), Rendall (2017), or Cerina et al. (2017). In terms of measuring the household sector production, some recent contributions are Bridgman (2016), Duernecker and Herrendorf (2017), or Bridgman, Duernecker, and Herrendorf (2017). However, none of these papers has been used to calculate the aggregate costs of gender inequality. Our goal in this paper is to build on Cuberes and Teignier (2016) and add a household sector to the model in the spirit of some of the papers just mentioned.

The rest of the paper is organized as follows. In Section 2 we present the theoretical framework. We show the parameter values and the numerical results in Section 3, while we study the effects of technology change in household durables in Section 4. Section 5 concludes.

2 Theoretical framework

In this section, we present the theoretical framework used to generate the quantitative predictions of Section 3, which is an extension of the model proposed by Cuberes and Teignier (2016). The details of the model solution are presented in the Appendix.

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4See also the reports by Goldman Sachs (2007), Aguirre et al. (2012), and McKinsey & Company (2015).
2.1 Setup description

The economy we consider has two sectors (market and household) that produce an homogeneous good, as well as a continuum of agents, indexed by their skill level \( x \), who own one unit of time. Talent here should be interpreted more broadly than in Lucas (1978) or Cuberes and Teignier (2016) since now it not only affects the entrepreneurs’ profits, but also the workers’ earnings.\(^5\) We assume the economy is closed, with an exogenous workforce of size \( P \). Skill-adjusted labor and capital are supplied by consumers to firms, in exchange for a wage rate per unit of skill, \( w \), and a capital rental rate, \( r \), respectively. These inputs are then combined by firms to produce a unique, homogeneous consumption. The stock of capital takes its steady-state value and, hence, its marginal product is equal to the depreciation rate plus the intertemporal discount factor.

Men choose to become either firm workers in the market sector, who earn the equilibrium wage rate \( w \) times their skill level \( x \), or entrepreneurs, who earn the profits generated by the firm they manage in the market sector. Women can also become workers or entrepreneurs but they also have the option of producing in the household sector. As in Lucas (1978) and Buera and Shin (2011), the production function of an employer is given by

\[
y(x) = x \left( k(x)^\alpha n(x)^{1-\alpha} \right)^\eta,
\]

where \( x \) denotes the talent or productivity level of the employer, \( n(x) \) is the units of skill-adjusted labor hired by the employer, \( k(x) \) is the units of capital rented by the employer, and \( y(x) \) represents the units of output produced. The parameter \( \eta \in (0, 1) \) measures the span of control of entrepreneurs and, since it is smaller than one, the entrepreneurial technology involves an element of diminishing returns. Since the price of the homogeneous good is normalized to one, employers’ profits are equal \( \pi(x) = y(x) - rk(x) - wn(x) \).

On the other hand, an agent with talent \( x \) who chooses to become self-employed in the market sector operates a technology given by

\(^5\)In what follows we will refer to an entrepreneur as someone who works as either an employer or a self-employed.
\[
\tilde{y}(x) = \tau x \left( \tilde{k}(x)^{\alpha} \tilde{n}(x)^{1-\alpha} \right)^{\eta}, \tag{2}
\]

where \( \tilde{k}(x) \) denotes the units of capital used and \( \tilde{y}(x) \) the units of output produced. \( \tilde{n}(x) = x \) are the skill-adjusted labor units the self-employed agents works in his or her own firm.\(^6\)

The parameter \( \tau \), which is calibrated to match the aggregate share of self-employed workers, captures the fact that self-employed agents have to spend some time on management tasks. Self-employed profits are equal to 
\[
\tilde{\pi}(x) = \tilde{y}(x) - r\tilde{k}(x).
\]

Finally, women can also produce in the household sector, operating the following technology:
\[
y_h = (A k_h + B n_h)^{\eta}, \tag{3}
\]

where \( k_h \) denotes the units of capital rent by the household sector and \( n_h \) the units of time allocated to the household sector. Note that this production function can be seen as the perfect substitutes version of the one in equation (1), with the productivity parameters \( A \) and \( B \) being independent of the agent talent. Women choose \( k_h \) and \( n_h \) in order to maximize their total earnings, which are given by their market-sector plus their household sector earnings.\(^7\)

Specifically, when the opportunity cost of time is their market wage \( w x \), women choose to allocate their unit of time in the household sector when 
\[
\frac{A}{B} < \frac{r}{wx},
\]

and they choose to allocate it to the market otherwise.\(^8\) Under this household production function, changes in the home technology parameter \( A \) (which can be interpreted as an increase in the availability of home appliances or the consumer durable goods revolution mentioned in Greenwood et al., 2005) lead to a rise of female labor participation, as in the model by Greenwood et al. (2005) which is empirically assessed by Cavalcanti and Tavares (2008).

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\(^6\)The consumption good produced by the self-employed and the capital they use is the same as the one in the employers’ problem. However, it is convenient to denote them \( \tilde{y} \) and \( \tilde{k} \) to clarify the exposition.

\(^7\)Arguably this is a unitary approach to the problem in the sense that a household in this model is effectively composed of only one person who can either be a man or a woman. A more realistic but complicated approach would recognize the importance of intra-household decisions as in Chiappori (1997). We leave this promising avenue for further research.

\(^8\)As explained in Appendix A, depending on the parameter values, women choosing to work at home may still want to rent some capital because their time endowment is limited. At the same time, there may be a group of women who allocate part of their time to the household sector and part of their time to the market sector.
2.2 Frictionless Equilibrium

In equilibrium, employers choose the units of labor and capital they hire in order to maximize their current profits, denoted by \( \pi_e \), while self-employed workers choose the units of capital to rent in order to maximize their profits, denoted by \( \pi_s \). Market workers earn a labor compensation equal to \( wx \). Women also choose the units of capital to rent for the household-sector production and the fraction of their time they want to allocate to this sector. If they choose to become full-time household workers, they earn an income denoted by \( \pi_{h0} \), while if they choose to become part-time household workers, they earn an income denoted by \( \pi_{h1} \), which includes market-sector earnings plus household-sector earnings.

The first plot of Figure 1 displays the payoff of the three market occupations at each talent level and shows the optimal occupational choices in equilibrium for men. Men with the highest skill level (those with talent above \( z_2 \)) become employers, whereas those with intermediate skill levels become self-employed. Finally, men with a level of talent lower than \( z_1 \) become market workers. The second plot of Figure 1 displays the slightly more complicated occupational map for women. As it was the case for men, women with talent above \( z_2 \) become employers, whereas those with talent between \( z_1 \) and \( z_2 \) choose to be self-employed. Women become market workers if their talent is between \( z_{f0} \) and \( z_1 \). Women with talent below \( z_{f0} \) allocate their time to the household sector production, either part time (between \( z_{f0} \) and \( z_1 \)) or full time (below \( z_{f0} \)).

In this economy, aggregate (market) production is the sum of output by male employers and male self-employed, as well as output by female employers and female self-employed:

\[
Y = N \left[ \int_{z_2}^{\infty} y(x) d\Gamma(x) + \int_{z_1}^{z_2} \bar{y}(x) d\Gamma(x) \right],
\]

where \( \Gamma(x) \) denotes the talent cumulative density function, which, again, it is assumed to be the same for men and women. The first term inside the bracket represents the production

\(^9\text{To be precise, } \pi_{h0} \text{ and } \pi_{h1} \text{ are defined here as the household production profits by household workers relative to market workers, who may also choose to engage in household production but using only capital.}\)
by male and female employers, whereas the second one is the corresponding term by self-employed.

Total production in the economy, $Y_T$, is the sum of market output ($Y$) and household output, $Y_h$.

$$Y_T = Y + Y_h.$$ 

$Y_h$ is equal to household production by full-time household workers, $y_{h00}$, plus household production by part-time household workers, $y_{h01}$, plus household production by female market workers, $y_{h1}$ (who use some capital in the household sector in order to produce there):

$$Y_h = \frac{N}{2} \left( \int_{z_{00}}^{z_{01}} y_{h00} d\Gamma(x) + \int_{z_{01}}^{z_{02}} y_{h01} d\Gamma(x) + \int_{z_{02}}^{\infty} y_{h1} d\Gamma(x) \right).$$

### 2.3 Introducing gender gaps into the framework

The model assumes that women are identical to men in terms of their innate skills but they face exogenous constraints in their market-sector occupational choice. These frictions may reflect discrimination, or other demand factors, but they might also reflect differences in optimal choices of women, or other supply factors. In this sense, our estimated effects should be interpreted as the result of all the factors that make women behave differently than men.
in the labor market.

The first constraint we impose is that females face a probability $\mu$ of being “allowed”\(^\text{10}\) to be an employer and a probability $1 - \mu$ of being excluded from employership. Out of the group of women not allowed to be employers, some have have the possibility of becoming self-employed while the rest are also excluded from self-employment. In particular, women excluded from employership have a probability $\mu_o$ of being allowed to be self-employed and a probability $(1 - \mu_o)$ of not being allowed to be self-employed. As a result a fraction $(1 - \mu)(1 - \mu_o)$ of women are shut out from entrepreneurship, i.e. both employership and self-employment, and can only become workers. Appendix B shows a graphical representation of the occupational choice of women taking into account the constraints just described.\(^\text{11}\) Finally, the third friction we introduce is that only a fraction $\lambda$ of women are allowed to participate in the labor market, while a fraction $(1 - \lambda)$ of randomly selected women are excluded from all the possible occupations in the labor market.\(^\text{12}\) In this setup, women who do not participate in the formal labor market become full-time workers in the household sector and, hence, the estimated aggregate income loss due to the $\lambda$ gender gap depends on the difference between the market participants earnings and the household-sector earnings.

The effects of the entrepreneurship gaps, $\mu$ and $\mu_o$, are illustrated in Figure 2 for the case without part-time workers. When some women are excluded from entrepreneurship, the supply of market workers increases, leading to a fall in the wage rate and a rise in the employers’ profit function. This makes both $z_1$ and $z_2$ fall, implying a lower average talent of entrepreneurs and a lower firm productivity. The capital stock adjusts downwards to keep its marginal product equal to the depreciation rate plus the intertemporal discount

\(^{10}\)Again, this constraint may represent either demand barriers, supply choices, or a combination of the two.

\(^{11}\)Note that, in this setup, we are not allowing for the possibility of women being excluded from self-employment but not from employership, since we think that whichever are the barriers women face to become self-employed, they should apply even more strongly to become an employer. In terms of the parameters of the model, if $\mu = 1$, then the value of $\mu_o$ does not affect the occupational choices of women.

\(^{12}\)We say that women excluded from the labor force are randomly selected because the talent of these excluded women is drawn from the same distribution as the rest of the population.
factor. Moreover, in the case of women, there is a rise in $z_0^f$, implying that the number of workers in the market-sector falls and the number of workers in the household sector rises. As a result of all these effects, market-sector output decreases. If part-time work was also considered, the fall in wages would lead to a rise of both $z_{00}^f$ and $z_0^f$, implying also a fall in female market work.

**Figure 2**: Qualitative effects of entrepreneurship gender gaps

The effects of the labor force participation gap, $\lambda$, are more straightforward. When some women get excluded from the market sector, they become household-sector workers, leading to a fall in the market-sector labor and a rise in the home-sector labor. As before, the capital stock adjusts downwards to keep its marginal product equal to the depreciation rate plus the intertemporal discount factor. These effects clearly reduce total output from the market sector, but they are likely to slightly increase output per worker because the household-sector capital demand falls and, thus, the market sector capital-to-labor ratio increases.

3 **Numerical results**

3.1 **Talent Distribution and Model Parametrization**

To simulate the model, we use a Pareto function for the talent distribution, as in Lucas
Table 2: Common parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.79</td>
<td>From Buera and Shin (2011)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.114</td>
<td>To match capital share: $\alpha \eta + (1 - \eta) = 0.3$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>6.3</td>
<td>To match the U.S. employer’s share</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.67</td>
<td>To match the U.S. self-employed share</td>
</tr>
<tr>
<td>$A$</td>
<td>0.14</td>
<td>To match the value of household output</td>
</tr>
<tr>
<td>$B$</td>
<td>0.86</td>
<td>To match the share of female part-time workers</td>
</tr>
</tbody>
</table>

(1978) and Buera et al. (2011). The cumulative distribution of talent is, hence, given by

$$\Gamma (x) = 1 - B^\rho x^{-\rho}, \ x \geq 0,$$

where $\rho, \ B > 0$.

The values used for the model parameters are showed in Table 2. The parameter $B$ of the talent distribution is normalized to 1, while the parameter $\eta$ is set to 0.79 as in Buera and Shin (2011).\(^{13}\) The capital-output elasticity parameter $\alpha$ is set to 0.114 in order to match the 30% capital income share observed in the U.S. data.\(^{14}\) The parameters ($\rho, \ \tau, \ A, \ B$) are estimated to match four different moments of the United States data. First, the fraction of employers in the labor force (which is 3.6%), second, the fraction of self-employed workers in the labor force (which is 6.5%), third, the household sector output relative to the market-sector one (which is 0.3), and, fourth, the gap between the share of female part-time workers and the share of male part-time workers (which is 10.6%). Data on employment status and working hours is obtained from the International Labour Organization, while data on household-
sector output is obtained from Bridgman et al (2017).

The values of the country-specific gender gaps \((\mu, \mu_0, \lambda)\) are computed to simultaneously match the female-to-male ratio of employers, self-employed workers, and labor market participation in each country. After matching these moments, we obtain that the average value of the employership gender gap, \(1 - \mu\), is 0.59, while the highest value is 0.78 and the lowest value is 0.38. For the self-employment gender gap, \((1 - \mu)(1 - \mu_0)\), the average value is is 0.45, the maximum is 0.68, and the minimum 0.08. Finally, for the labor force gender gap, \(1 - \lambda\), the average value is 0.15, the maximum is 0.46, and the minimum is 0.

3.2 Numerical results for European countries

The numerical results for the sample of 37 European countries, together with the United States, are summarized in Tables 3 and 4. Appendix C contains the results for all countries. As we can see in Table 3, the average fall in total output (market plus household) is similar in the baseline simulation (13.2%) and in the simulation with no household sector (12%). The rise in household sector production (5% in the baseline simulation) does not compensate for the fact that the fall in market output is much larger in the baseline simulation (17.5% vs. 12%). At the same time, the fall in female market sector hours is significantly larger in the baseline simulation than the in the simulation with no household sector (24.2% vs 14.6%), about half of which is due to entrepreneurship gender gaps in the case of the baseline simulation.

Intuitively, in the baseline simulation, the introduction of the gender gaps leads to an increase in the household sector output (5%), which obviously does not occur in the model without household sector, as we can see in the third rows of Table 3. At the same time, however, the market output loss due to the gaps is larger in the baseline simulation, since the fall in market output due to the entrepreneurship gender gaps in the baseline simulation is

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13Buera and Shin (2011) choose \(\eta\) to match the top five percent income share in the U.S., which is 30%. This is a reasonable approximation given that the top earners are entrepreneurs both in the model and the U.S. data.

14Entrepreneurs’ profits are considered capital income, thus we set \(\alpha\eta + (1 - \eta)\) equal to 30%.
Table 3: Average losses due to the gender gaps in Europe and the United States

<table>
<thead>
<tr>
<th>(% )</th>
<th>Baseline simulation</th>
<th>No household sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fall in market output due to entrepreneurship gaps</td>
<td>12.6</td>
<td>5.5</td>
</tr>
<tr>
<td>Fall in market output due to all gender gaps</td>
<td>17.5</td>
<td>12.0</td>
</tr>
<tr>
<td>Fall in household output due to all gender gaps</td>
<td>-5.0</td>
<td>0</td>
</tr>
<tr>
<td>Fall in total output due to all gender gaps</td>
<td>13.2</td>
<td>12.0</td>
</tr>
<tr>
<td>Fall in female mkt hours due to entrepreneurship gaps</td>
<td>12.0</td>
<td>0</td>
</tr>
<tr>
<td>Fall in female mkt hours due to all gender gaps</td>
<td>24.2</td>
<td>14.6</td>
</tr>
</tbody>
</table>

more than twice the one in the simulation without household sector (12.6% vs. 5.5%), as the first row of Table 3 shows. The reason is that, in the presence of a household sector, female labor supply endogenously falls after the introduction of the entrepreneurship gender gaps, as the fifth row of Table 3 shows. When faced with these barriers, a substantial number of women, some of them with high levels of talent, change occupations and, some, end up in the household sector. This then results in less able men managing firms, hence, generating a drag in the economy’s productivity.

Table 4 shows the average results by region: Eastern Europe, Northern Europe, Southern Europe, and Western Europe, plus the United States for comparison purposes.\textsuperscript{15} In the last column, we can see the lost in total output (market plus household) due to all the gender gaps considered, which is highest in Southern Europe (with 15.9%) and lowest in Eastern Europe (11.3%). The third column shows that rise in household-sector production. In the first two columns, we can see the effects of the gender gaps on market output per worker and market output per capita. The fall in market productivity (or market output per worker),

\textsuperscript{15}Eastern Europe is composed of Belarus, Bulgaria, Czech Republic, Hungary, Moldova, Poland, Romania, Russia, Slovakia, and Ukraine. Northern Europe is composed of Denmark, Estonia, Finland, Iceland, Ireland, Latvia, Lithuania, Norway, Sweden, and United Kingdom. Southern Europe is composed of Croatia, Cyprus, Greece, Italy, Macedonia, Malta, Portugal, Serbia, Slovenia, and Spain. Western Europe is composed of Austria, Belgium, France, Germany, Luxembourg, Netherlands, and Switzerland.
Table 4: Losses due to the gender gaps, by European region

<table>
<thead>
<tr>
<th>(%)</th>
<th>Market output per worker</th>
<th>Market output</th>
<th>Household output</th>
<th>Total output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eastern Europe</td>
<td>5.5</td>
<td>15.1</td>
<td>-4.8</td>
<td>11.3</td>
</tr>
<tr>
<td>Northern Europe</td>
<td>7.5</td>
<td>16.3</td>
<td>-5.4</td>
<td>12.1</td>
</tr>
<tr>
<td>Southern Europe</td>
<td>5.7</td>
<td>20.6</td>
<td>-3.8</td>
<td>15.9</td>
</tr>
<tr>
<td>Western Europe</td>
<td>6.6</td>
<td>18.4</td>
<td>-6.6</td>
<td>13.6</td>
</tr>
<tr>
<td>United States</td>
<td>6.3</td>
<td>17.3</td>
<td>-6.5</td>
<td>12.7</td>
</tr>
</tbody>
</table>

which is mainly due to the entrepreneurship gender gaps, is highest in Northern Europe, followed by Western Europe, and is lowest in Eastern Europe. This is contrast with the fall in total market output, which is highest in Southern Europe, indicating that the labor force participation gap is higher than in other regions. In conclusion, the lowest falls due to gender gaps are in Eastern Europe, while Northern Europe (and Western Europe, to a smaller extent) has a larger fall in productivity but a smaller fall in total market output and Southern Europe has a smaller fall in productivity but a larger fall in total market output. The United States has productivity losses comparable to Western Europe and market output losses comparable to Northern Europe.

3.3 Decomposition of the results

In this section we decompose the losses of market output into the three components of the production function. The first component, firm productivity, depends on the talent of employers and self-employed when capital and labor are kept constant. As explained in Section 2.3, the presence of the entrepreneurship gender gaps leads to a fall in the average talent of firm managers because less talented individuals replace those excluded by the gender gaps. The second component, the equilibrium capital stock, is negatively affected by introduction of gender gaps because both the fall in employment and the fall in firm productivity lead to a fall in the marginal product of capital. Finally, the third component, the equilibrium labor, is also negatively affected by the introduction of gender gaps, not only because of the direct effect of the $\lambda$-gap, but also because of the endogenous fall in the
female labor supply due to a market productivity drop.

The first three columns of Table 5 show the importance of each component for the four European regions as well as the United States. The contribution of firm productivity fall into the market output loss due gender gaps ranges from 23.2% in Southern Europe to 35.1% in Northern Europe. The contribution of the capital stock fall into the market output loss ranges from 5.7% in Northern Europe to 6.5% in Southern Europe, while the contribution of the labor input fall ranges from 59.3% in Northern Europe to 70.2% in Southern Europe. In conclusion, about two thirds of the market output loss is due the fall in the labor input, about one fourth is due to the fall in firm productivity, and the rest is due to the capital input fall.

Following Antunes et al. (2015), we next decompose the fall in output per worker into a TFP term and a capital intensity, term. Mathematically, if aggregate output per worker can be written as \( y = TFP \times k^\alpha \), then it must be also be the case that \( y = TFP^{1-\alpha} \left( \frac{k}{y} \right)^{\frac{\alpha}{1-\alpha}} \), where \( \frac{k}{y} \) is the capital-output ratio or capital intensity. The contribution of the capital intensity term is showed in column 4 of Table 5, and it corresponds approximately to one fourth of the fall in output per worker. The TFP term, on the other hand, explains about three fourth of the output per worker fall, as is showed in column 5 of Table 5. The contribution of the capital intensity term ranges from 18.2% in Northern Europe to 39.5% in Southern Europe, while the contribution of the TFP term ranges from 60.5% in Southern Europe to 81.8% in Northern Europe.

4 Extension: Effects of the “Engines of Liberation”

As discussed in Greenwood et al. (2005) and Cavalcanti and Tavares (2008), an important driver behind the secular rise in female labor supply is the introduction of household durable goods. This phenomenon was labeled as “engines of liberation” by Greenwood et al. (2005) and tested empirically by Cavalcanti and Tavares (2008). In this extension, we examine the effects of a 5% increase in the parameter \( A \) from equation 3, which corresponds to the productivity of capital in the household production function. We first look at the effects
on female market labor hours by European region when only the entrepreneurship gaps are present, when only the labor force participation gap is present, and when all gender gaps are present. Next, we study the output losses due to the introduction of the gender gaps under the new value of $A$.

The first three columns of Table 6 show that a 5% rise in $A$ leads to a rise of more than 11% in the market hours of women. Intuitively, when $A$ increases, more women find it optimal to increase their market participation and use more durables to produce in the household sector. This increase in female employment, however, would be much lower if the entrepreneurship gender gaps were not present, as we can see in column 3. When the entrepreneurship gender gaps are present, more women optimally decide to work in the household sector because the labor market wages are lower and, some of these women, reallocate to the market sector after the rise in $A$.

The last four columns of Table 6 show the output effects of introducing the gender gaps when $A$ has increased by 5%. Comparing these columns with Table 4, we can see that the effects on market output are now smaller, since less women decide to leave the market labor force. The increase in household output due to the gender gaps is now also smaller, since women working in the market sector are now available to produce more at the household using only capital. The fall in total output (market plus household) is now also smaller, due

<table>
<thead>
<tr>
<th>Table 5: Fraction of the loss due to each component, by European region (%)</th>
<th>Market output</th>
<th>Output per worker</th>
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<tr>
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<td>6.0</td>
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Table 6: Effects of a 5% increase in the parameter $A$, by European region

<table>
<thead>
<tr>
<th>Region</th>
<th>Rise in female labor hours</th>
<th>Output losses due to gender gaps</th>
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<tr>
<td></td>
<td>(%)</td>
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to the smaller drop in market output.

5 Conclusion

This paper uses a general equilibrium, occupational choice model with a household sector to examine the quantitative effects of gender gaps in entrepreneurship and workforce participation. The introduction of the household sector increases the estimated loss in market output, but it decreases the estimated loss in household output. Our simulations also show that gender gaps in entrepreneurship have very large negative effects in both income and aggregate productivity, since they reduce the entrepreneurs’ average talent as well as female labor force participation when we consider the possibility of working in the household sector. We then estimate the gender gaps for 37 European countries and we find that gender gaps cause an average market output loss of 17.5%, with Southern Europe being the region with the largest loss. About two thirds of the market output loss is due the fall in the labor input, about one fourth is due to the fall in firm productivity, and the rest is due to the capital input fall. The loss in total output, which also includes household production is 13%, slightly larger than the one predicted in a model without household sector. A rise in the household capital productivity has important effects on the equilibrium, leading to more female employment and lower income losses due to gender gaps, but we show that
this effect depends crucially on the gender gaps present in the economy. These results highlight the importance of accounting for household production in models that aim to quantify the effects of gender gaps in the labor market.
References


Economics 9(2), 508-523.


A  Model details

A.1  Agents’ optimization

A.1.1  Employers

Employers choose the units of labor and capital they hire in order to maximize their current profits $\pi$.

$$\max_{k,n} \left\{ x \left( k^{\alpha} n^{1-\alpha} \right)^\eta - rk - wn \right\},$$
The optimal number of workers and capital stock, \( n(x) \) and \( k(x) \) respectively, depend positively on the productivity level \( x \), as equations (5) and (6) show:

\[
n(x) = \left[ x\eta(1-\alpha) \left( \frac{\alpha}{1-\alpha} \right)^{\alpha \eta} \frac{w^{\alpha \eta - 1}}{r^{\alpha \eta}} \right]^{1/(1-\eta)}, \tag{5}
\]

\[
k(x) = \left[ x\eta \left( \frac{1-\alpha}{\alpha} \right)^{\eta (1-\alpha)} \frac{r^{\eta (1-\alpha) - 1}}{w^{\eta (1-\alpha)}} \right]^{1/(1-\eta)}. \tag{6}
\]

### A.1.2 Self-employed

When we solve for the problem of a self-employed agent with talent \( x \) who wishes to maximize his or her profits,

\[
\max_k \{ xk(x)^{\alpha \eta} - rk \},
\]

we find

\[
\tilde{k}(x) = \left( \frac{\tau x \alpha \eta}{r} \right)^{\frac{1}{r^{\alpha \eta}}}. \tag{7}
\]

### A.1.3 Household production

Women can get extra earnings from household production, hence they choose the household units of capital \( k_h \) and labor \( n_h \) in order to maximize their total earnings, which are given by their market-sector plus their household sector earnings. Specifically, when their optimal occupational choice in the market is to become a worker, their optimization problem is

\[
\max_{k_h, n_h} \left\{ (Ak_h + Bn_h)^{\eta} + wx \left( 1 - n_h \right) \right\},
\]

with \( n_h \in [0, 1] \) and \( k_h \geq 0.16 \) As a result, when \( \frac{A}{B} > \frac{r}{wx} \), women choose to allocate all their time to the market sector and rent \( k_h^1 \equiv \left( \frac{n_h \eta}{r} \right)^{\frac{1}{1-\eta}} \) units of capital. When \( \frac{A}{B} < \frac{r}{wx} \), on the

\[16\text{Note that if a woman is an employer or a self-employed, it will never be optimal for her to spend some time in household production.} \]
other hand, women allocate at least part of their time endowment to the household sector. In particular, their optimal time allocation to the household sector is \( n_h^0 = \min \left\{ 1, \left( \frac{\eta B^n}{w x} \right)^{\frac{1}{1-\eta}} \right\} \), which implies that some women with high market productivity may choose to allocate part of their time to the household sector and part of their time to the market sector. Women supplying all their labor to the market sector choose to rent \( k_h^0 = \max \left\{ 0, \left( \frac{\eta A^n}{r} \right)^{\frac{1}{1-\eta}} - \frac{B}{A} \right\} \) units of capital.

In other words, when \( \frac{rB^{1-\eta}}{\eta A} < 1 \), women choose their labor allocation as follows:

\[
n_h = \begin{cases} 
0 & \text{if } x > \frac{B}{A} \frac{r}{w} \\
1 & \text{otherwise}
\end{cases}
\]  
(8)

and their units of capital used in the household sector are equal to

\[
k_h = \begin{cases} 
\left( \frac{\eta A^n}{r} \right)^{\frac{1}{1-\eta}} & \text{if } x > \frac{B}{A} \frac{r}{w} \\
\left( \frac{\eta A^n}{r} \right)^{\frac{1}{1-\eta}} - \frac{B}{A} & \text{otherwise}
\end{cases}
\]  
(9)

producing the following units of output:

\[
y_h = \left( \frac{\eta A}{r} \right)^{\frac{n}{1-\eta}}
\]  
(10)

in both cases.

On the other hand, when \( \frac{rB^{1-\eta}}{\eta A} > 1 \), women choose their labor allocation as follows:

\[
n_h = \begin{cases} 
0 & \text{if } x > \frac{B}{A} \frac{r}{w} \\
\left( \frac{\eta B^n}{w x} \right)^{\frac{1}{1-\eta}} & \text{if } \frac{\eta B^n}{w} < x < \frac{B}{A} \frac{r}{w} \\
1 & \text{if } x < \frac{\eta B^n}{w}
\end{cases}
\]  
(11)
and their units of capital used in the household sector are equal to

\[ k_h = \begin{cases} 
\left( \frac{n_A}{r} \right)^{\frac{1}{1-\eta}} & \text{if } x > \frac{B}{A} \frac{r}{w} \\
0 & \text{otherwise} 
\end{cases} \quad (12) \]

producing the following units of output:

\[ y_h = \begin{cases} 
\left( \frac{n_A}{r} \right)^{\frac{\eta}{1-\eta}} & \text{if } x > \frac{B}{A} \frac{r}{w} \\
\left( \frac{n_B}{w} \right)^{\frac{\eta}{1-\eta}} & \text{if } \frac{n_B}{w} < x < \frac{B}{A} \frac{r}{w} \\
B^n & \text{if } x < \frac{n_B}{w} 
\end{cases} \quad (13) \]

### A.1.4 Occupational choice

Figure (1) displays the shape of the profit functions of employers \((\pi_e(x))\) and self-employed \((\pi_s(x))\) along with wage function earned by employees and the female household workers extra earning as a function of talent \(x\).\(^{17}\) The figure also shows the relevant talent cutoffs for the occupational choices. Here we present the equations that define the three thresholds. The threshold, \(z_1\), determines the earnings such that agents are indifferent between becoming workers or self-employed and it is given by

\[ wz_1 = \tau z_1 \hat{k}(z_1)^{\alpha\eta} - r \hat{k}(z_1). \quad (14) \]

If \(x \leq z_1\) agents choose to become workers, while if \(x > z_1\) they become self-employed or employers. The cutoff, \(z_2\), on the other hand, determines the choice between being a self-employed or an employer and it is given by

\[ \tau z_2 \hat{k}(z_2)^{\alpha\eta} - r \hat{k}(z_2) = z_2 x \left( k(z_2)^\alpha n(z_2)^{1-\alpha} \right)^\eta - r k(z_2) - wn(z_2) \quad (15) \]

\(^{17}\)In order to construct this figure we are implicitly using parameter values such all occupations are chosen in equilibrium and that part-time work is not optimal.
so that if \( x > z_2 \) an agent wants to become an employer.

Finally, the cutoff \( z^f_0 \), defines the talent level at which women are indifferent between being household workers, who only get earnings from their household production, and market workers, who get wage income plus household income from the household capital production. Specifically, when \( \frac{r^{B^1-\eta}}{\eta A} < 1 \), household workers get earnings \( r \left( \left( \frac{\eta A}{r} \right)^{\eta \eta} \right) \), while market workers get their wage income plus household earnings equal to \( r \left( \frac{\eta A}{r} \right)^{\eta \eta} - \left( \frac{\eta A}{r} \right)^{1-\eta} \). Hence, the difference between the household sector earnings is equal to \( r \frac{B}{A} \) and the talent threshold \( z^f_0 \) is defined as

\[
\frac{r B}{A} = w z^f_0.
\] (16)

Therefore, if their talent is below \( z^f_0 \), women maximize their earnings as household workers, while above \( z^f_0 \) their earnings are maximized as market workers.

When \( \frac{r^{B^1-\eta}}{\eta A} > 1 \), on the other hand, there are some women working full time in the household sector, some working part-time in the household sector and part-time in the market sector, and some other women working full time in the market sector. Women with ability below \( z^f_{00} \), where \( z^f_{00} \equiv \frac{nB^0}{w} \), choose to work full time in the household sector, and earn \( B^0 \). Women with ability between \( z^f_{00} \) and \( z^f_0 \), where \( z^f_0 \) is defined in equation (16), choose to allocate part of their time to the market and part of their time to the household. Their total earnings are \( \left( \frac{\eta B}{wx} \right)^{\eta \eta} \) from the household production plus \( wx \left( 1 - \left( \frac{nB^0}{wx} \right)^{1-\eta} \right) \) from the market sector, compared to total earnings of \( wx + \left( \frac{\eta A}{r} \right)^{\eta \eta} - r \left( \frac{\eta A}{r} \right)^{1-\eta} \) by female workers.

When \( \frac{r^{B^1-\eta}}{\eta A} > 1 \) women have actually five occupational choices, since some choose to work part time in the market and part time in the household sector. In this case, the earning functions are defined as

\[
\pi^0_{h} \equiv B^0 - \left( 1 - \eta \right) \left( \frac{\eta A}{r} \right)^{\eta \eta}
\]

and

\[
\pi^1_{h} \equiv wx + \left( 1 - \eta \right) \left( \left( \frac{\eta B}{wx} \right)^{\eta \eta} - \left( \frac{\eta A}{r} \right)^{\eta \eta} \right),
\]

which correspond to the household workers earnings minus the household production.
earnings of female market workers.

A.2 Competitive Equilibrium in a model with household sector

We assume that women represent half of the population in the economy and that there is no unemployment. Moreover, any agent in the economy can potentially participate in the labor market, except for the restrictions on women described above. Under these assumptions, in equilibrium, the total demand of capital by employers and self-employed must be equal to the aggregate capital endowment (in per capita terms), $k$:

$$k = \frac{1}{2} \left[ \int_{z_1}^{\infty} k(x) d\Gamma(x) + \int_{z_1}^{\infty} \tilde{k}(x) d\Gamma(x) \right] + \frac{\lambda}{2} \left[ \int_{z_2}^{\infty} \mu k(x) d\Gamma(x) + \int_{z_1}^{\infty} (\mu + (1 - \mu)k_0) \tilde{k}(x) d\Gamma(x) + \int_{z_2}^{\infty} (1 - \mu)k_0 \tilde{k}(x) d\Gamma(x) \right]$$

(17)

$$+ \frac{\lambda}{2} \left[ \int_{z_2}^{\infty} k_h^0 d\Gamma(x) + \int_{z_1}^{\infty} k_h^1 d\Gamma(x) \right] + \frac{1 - \lambda}{2} \int_{z_2}^{\infty} k_h^0 d\Gamma(x).$$

The first line of equation (17) is the demand for capital by men, while the two lower lines are the women’s demand for capital. The demand for capital by male-run firms has two components: the first one represents the capital demand by employers, while the second represents the demand by self-employed.

The demand of capital by women has six components, the first three corresponding to the market-sector firms run by women and the last three corresponding to the household-sector capital. The first one represents the capital demand by female employers, i.e. those with enough ability to be employers and who are allowed to be so, while the second term represent the capital demand by women who have the right ability to be self-employed. The third term shows the capital demand by women who become self-employed because they are excluded from employership. The fourth term corresponds to the household-sector capital demand by women who choose to be household-sector workers, the fifth is the household-sector capital demanded by women supplying the entire labor supply to the
market sector, and the last term is the household-sector capital demand by women who work in the household-sector because they are not allowed to work in the market sector.

Similarly, the labor market-clearing condition is given by

$$\frac{1}{2} \left[ \int_{z_2}^{\infty} n(x) d\Gamma(x) \right] + \frac{\lambda}{2} \left[ \int_{z_2}^{\infty} \mu(x) n(x) d\Gamma(x) \right] =$$

$$\frac{1}{2} \int_{z_1}^{z_2} xd\Gamma(x) + \frac{\lambda}{2} \left[ \int_{z_1}^{z_2} xd\Gamma(x) + \int_{z_1}^{\infty} ((1 - \mu) (1 - \mu_0)) xd\Gamma(x) + \int_{z_1}^{\infty} x (1 - n_h^0 (x)) d\Gamma(x) \right],$$

where the first line represents the skill-adjusted aggregate labor demand and the second line represents the skill-adjusted aggregate labor supply in the market sector. The aggregate labor demand is equal to the male employers demand (first term) and the female employers demand (second term), i.e. those women with enough ability to be employers who are allowed to choose their occupation freely. The aggregate labor supply is equal to the male workers supply (first term in second line) plus the female workers supply (second, third, and fourth term in second line). The female workers supply is given by the skill-adjusted labor of women who, given their talent, choose to be full-time workers, plus that of women who have enough ability to be employers or self-employed but are excluded from both occupations. Finally, some women working in the household sector may also choose to be part-time workers in the market sector.

A competitive equilibrium in this economy is a set of cutoff levels \((z_f^{h0}, z_f^{h1}, z_1, z_2)\), a set of quantities \([n(x), n_h^0 (x), k (x), k_h^0, k_h^1] \), \(\forall x\), and prices \((w, r)\) such that entrepreneurs choose the amount of capital and labor to maximize their profits, and labor and capital markets clear.
B  Women occupational choice map

C  Country-by-country results: long run income losses from labor market gender gaps

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<tr>
<th>%</th>
<th>Baseline simulation</th>
<th>No household sector simulation</th>
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<td></td>
<td>Loss in Y/N due to all gender gaps</td>
<td>Loss in Y/P due to gender gaps</td>
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