Family Labor Market Decisions and Statistical Gender Discrimination

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Empirical evidence

- Average gender wage gap still around 20%.
  (source: United States, CPS)

- About half of it not explained by human capital or occupation differences. (source: Blau and Kahn, 2017)
Gender wage gap within couples is larger for couples with children. (source: United States, CPS)
Empirical evidence (3)

- Also the case for gap in labor force participation.
  (source: United States, CPS)
Empirical evidence (4)

- Similar conclusion from time series evidence. (source: United States 1984-2014, PSID)
This paper:

- We propose a labor market framework with imperfect information that gives rise to statistical gender discrimination.

- We characterize the equilibrium and study:
  - Degree of misallocation due to gender discrimination.
  - Effects of changes in assortative mating and policy interventions.

Model intuition:

- When children arrive, couples jointly decide which member quits job based on their wages.

- Firms are interested in long relationships, generating "efficiency wages" mechanism.

- If women are expected to quit with higher probability, they get lower wage; initial beliefs get self-fulfilled.
Related Literature

  In equilibrium, only one high-earner in each household and, thus, they can enjoy within household specialization.

- Albanesi and Olivetti (2009).
  If firms believe home hours are higher for women, firms will offer them labor contracts with lower earnings, performance pay, and effort.

- Dolado, Garcia-Peñalosa, and De la Rica (2013).
  If women are expected to do more effort in the household, firms choose to invest less on them and pay them lower wages.

  If women, on average, have higher participation costs, employers will expect them to have higher turnover rate than men and will pay them lower wages. This channel quantitatively accounts for large fraction of gender wage gap and its decline.

Our setting: endogenous efficiency wages, emphasis on assortative mating.
Plan of the talk

1. Introduction.

2. Simplest example (unique productivity).

3. Heterogenous-productivity model:
   1. Setup description.
   2. Gendered equilibrium characterization.
   3. Non-gendered equilibrium characterization.


5. Concluding remarks and future work.
SIMPLEST EXAMPLE
Households:

- One male and one female; identical in all dimensions except gender.
- All agents have common productivity $y$.
- Couples hit by child shock with probability $\lambda$.
- Joint labor decision: when child shock hits, member with lower wage quits job (50% probability if wages are equal).

Firms:

- Hiring cost: $c$.
- Value of firm with female worker, $J$, and male worker, $K$:

$$J = (y - w) p_f (\text{stay at work—}w) - c$$
$$K = (y - s) p_m (\text{stay at work—}s) - c$$

where $w$ denotes female wage and $s$ male wage.
Under perfect competition, an equilibrium is a wage pair \((w, s)\) s.t.
- all firms have 0 profits,
- there is no profitable deviation for firms.

**Lemma**

There exists no (pure strategy) equilibrium without discrimination.

**Proof.**

If \(w = s = y - \frac{c}{1 - \lambda / 2}\), increasing the wage marginally is a profitable deviation:

\[
y - (w + \varepsilon) - c > (y - w) \left(1 - \frac{\lambda}{2}\right) - c = 0
\]
If gender is the only observable characteristic, there are two equilibria:

- The “female discrimination” equilibrium:
  \[ s = y - c, \quad w = y - \frac{c}{1 - \lambda}. \]

- The “male discrimination” equilibrium:
  \[ s = y - \frac{c}{1 - \lambda}, \quad w = y - c. \]

Note: if \( y - \frac{c}{1 - \lambda} < 0 \), discriminated agents stay out of the labor force.
“Beauty” Equilibrium

- Assume firms can’t/don’t discriminate based on gender but there is another observable characteristic.

- Beauty: \( b \sim U[0, 1]. \)

- Husband and wife draw iid draws; firms observe employees’ \( b \) but not partners’.

- In equilibrium, couple member with highest \( b \) stays:

  \[
  J(b) = (y - w(b))(1 - \lambda + \lambda b) - c = 0.
  \]

  Hence,

  \[
  w(b) = y - \frac{c}{1 - \lambda (1 - b)}.
  \]

- Note: \( b = 1 - \frac{y-c}{\lambda y} \) is lowest-\( b \) participant.
Gendered equilibrium:

\[ Y_g = y - c \]

“Beauty” equilibrium:

\[ Y_{beauty} = 2 \left( y \int_0^b \left( 1 - \lambda + \lambda \int_0^{b'} dF(b') - c \right) dF(b) \right) = \frac{(y - c)^2}{\lambda y} < Y_g \]

No discrimination policy (CCCP policy):

\[ Y_{policy} = 2y \left( 1 - \frac{\lambda}{2} \right) - 2c = (2-\lambda)y - 2c < Y_{beauty} \]

\[ Y_{policy} < Y_{beauty} < Y_g \]
HETEROGENEOUS-AGENTS MODEL
Couples get productivity draw pair \((x, y)\) from a joint productivity distribution function.

\[ x \sim U[0, 1], \quad y \sim U[0, 1]. \]

Firms observe employees’ \(y\) but not partners’.

\[ x|y = y \text{ with probability } \rho, \quad x|y \sim U[0, 1] \text{ with probability } 1 - \rho: \]

\[
F(x|y) = \begin{cases} 
(1 - \rho) x & \text{if } x < y \\
\rho + (1 - \rho) x & \text{if } x \geq y 
\end{cases}
\]
**Definition**

A pure-strategy equilibrium is a set of functions $w(x)$ and $s(y)$ that associates a single wage to each productivity such that

1. Firms make zero expected profit.
2. There is no higher wage that makes strictly positive profits.
3. When baby shock hits, workers leave if their wage is strictly lower than partners’; leave with 50% probability if wages are equal.

**Firms’ profit functions (case $\lambda = 1$):**

\[
J(w|x) = (x - w) \left( \text{Prob}(y < y(w) | x) + \frac{1}{2} \text{Prob}(y = y(w) | x) \right) - c
\]

\[
K(s|y) = (y - s) \left( \text{Prob}(x \leq x(s) | y) + \frac{1}{2} \text{Prob}(x = x(s) | y) \right) - c
\]
Equilibrium description ($\lambda = 1$ case)

Lemma

When $\rho > 0$, there does not exist a pure-strategy symmetric equilibrium (i.e. $x(w) \neq y(w)$).

Proof.

If $x(\cdot) = y(\cdot)$, increasing the wage marginally is a profitable deviation:

$$J(w, x) = (x - w) \left( \int_0^{y(w)} dF(y|x) + \frac{1}{2} \text{Prob}(y = y(w)|x) \right) - c$$

$$= (x - w) \left( (1 - \rho) y(w) + \frac{1}{2} \rho \right) - c$$

$$< (x - (w + \varepsilon)) \left( (1 - \rho) y(w) + \rho \right) - c.$$
**Gendered equilibrium: description (λ = 1 case)**

**Lemma**

*There exists a pure strategy equilibrium where* \( y(w) < x(w) \).

- In this “male-favoring” equilibrium
  
  \[
  J(w, x) = (x - w)(1 - \rho)y(w) - c \quad \forall w \\
  K(s, y) = (y - s)((1 - \rho)x(s) + \rho) - c \quad \forall s
  \]

- Imposing that \( J(w, x(w)) = 0 \) and \( K(w, y(w)) = 0 \),
  
  \[
  x(w) = w + \frac{c}{(1 - \rho)y(w)} \\
  y(w) = w + \frac{c}{\rho + (1 - \rho)x(w)}
  \]

- Note: in this region, when \((x, y)\) are active in the market,
  
  \[
  \frac{dx}{dw} < 1, \quad \frac{dy}{dw} < 1 \quad \Rightarrow \quad \frac{dw}{dx} > 1, \quad \frac{dw}{dy} > 1
  \]
Gendered equilibrium: characterization ($\lambda = 1$ case)

- Let $w^*$ define the highest wage paid to women.
  - $x^* = 1$.
  - $(1 - w^*)(1 - \rho)(w^* + c) - c = 0$.
  - $y^* = w^* + c$.
  - $w = y - c \forall y \geq y^* \Rightarrow \frac{dw}{dy} = 1 \forall y \geq y^*$.

- Let $\tilde{w} > 0$ be the lowest wage paid to women and $\tilde{x}$ the productivity of the lowest paid woman.
  - $\frac{\partial J(w, \tilde{x})}{\partial w} = 0 \Rightarrow \frac{dx}{dw} = 0 \Rightarrow \tilde{y} = \sqrt{\frac{c}{1-\rho}}$.
  - $\tilde{x} = \tilde{w} + \sqrt{\frac{c}{1-\rho}}$.
  - $y = w + \frac{c}{\rho + (1-\rho)\tilde{x}} \forall y < \tilde{y} \Rightarrow \frac{dw}{dy} = 1.$
Gendered equilibrium: characterization (2)

\[ w^* \]
\[ \tilde{w} \]

Segment I
Segment II
Segment III
Changes in degree assortative mating

- The more assortative mating, the more the market separates males and females:

![Labor Force Participation](image1)

![Avg gender wage gap (cond. on productivity)](image2)
A symmetric equilibrium is a set of cumulative wage distribution functions $F(w|x)$, $\forall x$, and $H(s|y)$, $\forall y$, such that

1. $F(w|x) = H(s|y)$.

2. Firms make zero expected profit in equilibrium:

$$J(w, x) = (x - w)(\rho F(w|x) + (1 - \rho) G(w)) - c = 0 \forall x, \forall w \in \{w(x), \tilde{w}(x)\}$$

where $G(w) = \int_0^1 F(w|\tilde{x}) d\tilde{x}$.

3. $J(\tilde{w}, x) < 0 \forall \tilde{w} \notin suppF(w|x)$.

4. After the baby-shock, the female worker leaves if $w < s$, stays if $w > s$, and leaves with 50% probability if $w = s$. 
Non-gendered equilibrium: characterization ($\lambda = 1$ case)

Let’s define two functions $\underline{x}(w)$ and $\overline{x}(w)$ such that

1. $\underline{x}(w)$ is the lowest productivity that a worker can have and still be paid $w$:
   \[
   (\underline{x}(w) - w)(\rho + (1 - \rho)G(w)) - c = 0.
   \]

2. $\overline{x}(w)$ is the highest productivity that a worker can have and still be paid $w$:
   - In segment III, $\overline{x}(w) = 1$.
   - In segment II, $(\overline{x}(w) - w)(1 - \rho)G(w) - c = 0$.
   - In segment I,
     \[
     (\overline{x}(w) - w)(\rho U(\overline{x}(w)) + (1 - \rho)G(w)) - c = 0.
     \]
Non-gendered equilibrium: characterization

\[ \bar{x}(1 - c) = 1 \]

\{ Segment III \}

\{ Segment II \}

\{ Segment I \}
Equilibria comparison: earnings dispersion

Earning Functions
\((\lambda=0.95, c=0.1, \rho=0.1)\)

- Gendered Eq: male earnings
- Gendered Eq: female earnings
- Non-gendered Eq: \(\bar{w}(x)\)
- Non-gendered Eq: \(\bar{w}(x)\)

Earning Functions
\((\lambda=0.95, c=0.1, \rho=0.7)\)

- Gendered Eq: male earnings
- Gendered Eq: female earnings
- Non-gendered Eq: \(\bar{w}(x)\)
- Non-gendered Eq: \(\bar{w}(x)\)
Welfare Analysis: Gendered Equilibrium

**Net output under the gendered equilibrium:**

\[ Y = \int_{y}^{1} (y p_m(y) - c) \, dy + \int_{x}^{1} (x p_f(x) - c) \, dx \]
Welfare Analysis: Social Planner

- **Net output under the Social Planner:**

  - If $y \leq c$: never employed.
  
  - If $c < y \leq y^s$: employed if better than spouse, where
    \[ y^s (1 - \lambda) - c = 0. \]
  
  - If $y > y^s$: always employed (if hit by kid shock, remains at work if better than spouse).

\[
Y^s = 2 \int_c^{y^s} (y - c) p_1(y) \, dy + 2 \int_{y^s}^1 (yp_2(y) - c) \, dy
\]
Welfare Analysis: CCCP policy

- Net output under No-Discrimination Policy (CCCP policy):
  
  - If $y > y^p$ : always employed, where

  $$ y^p \left( 1 - \lambda + \lambda (1 - \rho) y^p + \lambda \rho \frac{1}{2} \right) - c = 0 $$

  (if hit by kid shock, remains at work if better than spouse).

  $$ Y^p = 2 \int_{y^p}^{1} (y^p(y) - c) \, dy $$
Welfare Analysis: comparison

Net Surplus
($\lambda=0.95, c=0.1$)

First Best
Gendered Equilibrium
Non-gendered Equilibrium
CCCP Policy
MODEL EXTENSIONS
(work in progress)
Model Extensions: gender differences

1. Different distribution of market productivity.

- “Market productivity” of a woman with rank $\hat{x}$: $x = \delta \hat{x}$, where $\delta < 1$ and $\hat{x} \sim U[0, 1]$.

- Probability to marry a man with equal rank: $\rho$.

2. Different home productivity.

- Women have “home productivity advantage” with babies.

- Women stay at home with baby if $w < s + \phi$.

- Hence, some women with higher wage than husband would leave.

Note: if $\delta < 1$ or $\phi > 0$, women are less likely to stay at work; hence, there is “gender discrimination” even if $\rho = 0$. 
Productivity differences: equilibrium firm profits

Firm profit functions:

\[ J(w|x) = (x - w) \ P(y(w - \phi)) - c \ \forall w \]
\[ K(s|y) = (y - s) \ Q(x(s + \phi)) - c \ \forall s \]

where

\[ P(y(w - \phi)) = (1 - \lambda) + \lambda (1 - \rho) \ y (w - \phi) \]
\[ Q(x(w + \phi)) = (1 - \lambda) + \lambda \left[ \rho + (1 - \rho) \ \frac{x(w + \phi)}{\delta} \right] \]
Productivity differences: equilibrium characterization

- $\exists w^* : x(w^*) = \delta$ and $\forall w > w^*$ there exists no female with productivity high enough to get $w$.
  - The man that makes the family indifferent has salary $s^* = w^* - \phi$ and always stays in work:
    $$K(w^* - \phi, y^*) = (y^* - w^* + \phi) - c = 0$$
  - The woman with wage $w^*$ stays in work if husband has productivity lower than $y^*$:
    $$J(w^*, \delta) = (\delta - w^*)((1 - \lambda) + \lambda(1 - \rho)y^*) = 0$$

- $\exists \tilde{w} : \frac{\partial J(w, \tilde{x})}{\partial w} = 0$ and $\forall w < \tilde{w}$ there exists no female with productivity low enough to get $w$.

- $\forall w \in [\tilde{w}, w^*]$ there are a pair of male and female productivity $x$ and $y$ such that the following two expressions hold:
  $$J(w, x) = (x - w)P(y) - c = 0$$
  $$K(w - \phi, y) = (y - w + \phi)Q(x) - c = 0$$
Gender Gap in Labor Force Participation

Labor force participation: female-to-male ratio
($\lambda=0.95$, $c=0.1$)

- Blue line: ($\delta=1$, $\phi=0$)
- Red line: ($\delta=0.9$, $\phi=0$)
- Yellow line: ($\delta=1$, $\phi=0.1$)
Gender Wage Gap

- Average wage gap conditional on productivity (unobserved):

![Graph showing the average gender wage gap](image)

\[ \text{Average gender wage gap} \]

\( (\lambda = 0.95, c = 0.1) \)
Gender Wage Gap (2)

- Observed average gender wage gap:

![Graph showing the average gender wage gap](image)

**Average gender wage gap**

\[
(\lambda=0.95, \ c=0.1)
\]

- \( (\delta=1, \ \phi=0) \)
- \( (\delta=0.9, \ \phi=0) \)
- \( (\delta=1, \ \phi=0.1) \)
Male Wage Inequality

- Ratio of the 90/50 wages in the population productivities:

![Male wage inequality: 90-50 ratio](image)

Male wage inequality: 90-50 ratio

\((\lambda=0.95, c=0.1)\)
**Male Wage Inequality (2)**

- Ratio of the 90/50 wages in the working population:

![Graph showing Male wage inequality: 90-50 ratio](image_url)
Concluding remarks

- In our framework,
  - there is contracting friction between husband and wife employers;
  - under assortative mating, firms want to discriminate and pay some workers “efficiency wages” to rise probability they stay.

Main results:
- If women get discriminated, they are paid less at all productivities.
- High-productivity women *always* employed but paid less than men; low-productivity women are not employed.
- Assortative mating reduces female participation and increase the gender wage gap at each productivity level.
- Female participation increases male wage inequality.
- Gender discrimination creates misallocation and reduces aggregate output; non-discrimination policy does not help.
Future work

- Theoretical analysis when differences by gender:
  What is the equilibrium with anti-discrimination laws when there are productivity differences across genders? What if laws are only binding for a subset of firms?

- Numerical analysis:
  Calibrate extended model to get quantitative predictions on evolution gender gaps and effects of policy interventions.

- Model with education decisions (endogenous $\delta$):
  What are the effects of labor market discrimination on incentives to accumulate skills? Is there a “discrimination multiplier”? 