Abstract

We develop a multi-sector spatial equilibrium model with endogenous land use: land is used either for agriculture or housing. Urban land, densely populated due to commuting frictions, expands out of agricultural land. With rising productivity, the reallocation of workers away from agriculture frees up land for cities to expand, limiting the increase in land values despite higher income and increasing urban population. Due to the reallocation of land use, the area of cities expands at a fast rate and urban density persistently declines, as in the data over a long period. As structural change slows down, cities sprawl less and land values start increasing at a faster rate, as in the last decades. Quantitative predictions of the joint evolution of density and land values across time and space are confronted with historical data assembled for France over 180 years.

Keywords: Structural Change, Land Use, Productivity Growth, Urban Density.

JEL-codes: O41, R14, O11

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1 Introduction

Since the early years of the industrial revolution, the population massively migrated from rural areas towards cities. This widespread phenomenon of urbanization went together with the reallocation of workers away from the agricultural sector towards manufacturing and service sectors—a phenomenon of structural change. How do cities grow when these well-known phenomena occur? Cities can become denser for a given area—growth at the intensive margin. They can also become larger in surface to accommodate more workers—via growth at the extensive margin. Over a long period, cities have been growing essentially in area, at such a fast speed that their average density has been falling. In other words, over time, cities expanded faster in area than in population. We precisely document this stylized fact for France since 1870 but it is also documented on a global scale in Angel et al. (2010). In France, the population of the main cities has been multiplied by almost 4 since 1870, while their area increased by a factor 30: the average urban density has thus been divided by about 8. This paper shows that this persistent decline in density, despite the process of urbanization, is well explained by the most conventional theories of structural change with non-homothetic (Stone-Geary) preferences and augmented with endogenous land use—whereby land can be used for agriculture or urban housing.

A crucial insight of our theory is to consider that the value of agricultural land at the urban fringe determines the opportunity cost of expanding the area of cities for housing purposes. With low agricultural productivity, agricultural goods and farmland are expensive. High agricultural land values make cities initially small in area and very dense as households cannot afford large homes—a manifestation of the ‘food problem’ (Schultz (1953)). With structural change driven by rising (agricultural) productivity, workers move away from rural areas towards cities, freeing up agricultural land. As the value of land at the urban fringe falls and households free up resources to buy larger homes, cities expand in area at a fast rate. Together with the reallocation of workers across sectors, reallocation of land use occurs—from agricultural use to urban use. We document that for France, since 1840, about 15% of French land formerly used for agriculture is no longer used for this purpose. As long as the transitory process of reallocation away from agriculture continues, cities grow faster in area than population and average urban density keeps falling with urban expansion. Thus, our theory provides a novel mechanism explaining the sprawl and the suburbanization of cities. This complements the traditional urban view that cities have sprawled following improvements in the commuting technologies that have allowed households to live further away from their workplace.

Our framework also provides novel predictions regarding the historical evolution of land values. When productivity is low and agricultural goods are in high demand for subsistence needs, the value of farmland is high relative to income. With economic development, structural change frees up farmland for urban expansion and puts downward pressure on its price. The value of agricultural land as share of income falls and, over time, the value of urban land constitutes the largest fraction of aggregate land values. These predictions are in line with the data as shown in Piketty and Zucman.
Moreover, despite rising housing demand, the fast expansion of cities at the extensive margin due to structural change initially limits the increase in urban land rents and housing prices. When the reallocation of workers slows down, so does the reallocation of land use at the fringe of cities. If workers’ productivity increases further, the value of land must adjust to prevent further expansion of cities with rising housing demand. Land values start to increase at a faster rate. Our theory thus predicts flat land and housing values for decades before shooting up as the process of structural change ends. This prediction resembles very much the data for France and most advanced economies as best illustrated in Knoll et al. (2017): real housing prices being flat for decades since the nineteenth century before increasing at a fast rate in the recent decades—a hockey-stick pattern of housing prices and land values. Therefore, our theory provides novel insights on the joint evolution of the density of cities and land values along the process of economic development. It also helps understanding how the structure of cities, e.g. their urban extent and density evolves with the process of structural transformation. It sheds new light on the origins of urban sprawl in the process of economic development—a central matter in the artificialization of soils and their environmental impact (IPCC (2018)).

The contribution of our paper is threefold. First, we document new stylized facts on land use and urban expansion for France since the mid-nineteenth century. In particular, using historical maps and satellite data for the more recent period, we document the historical decline of the density of French cities. Between 1870 and 1950, the average density was divided by about 3 and again by about 2.5 until 1975—the thirty years post-World War II being characterized in France by a faster structural change and rural exodus (Mendras (1970), Bairoch (1989), Toutain (1993)). Together with the slowdown of structural change in the more recent decades, average urban density did not fall much since. These facts, together with the historical evolution of urban and agricultural land values in France, motivate our theory.

The second contribution is to develop a spatial general equilibrium model of structural change with endogenous land use—agricultural or residential land use. The production side features three sectors: rural, urban and housing. The rural (resp. urban) sector produces agricultural (resp. non-agricultural) tradable goods, the production of the rural good being more land intensive. The housing sector produces location-specific housing units using the urban good and land in the process. Land is in fixed supply and land use rivalrous: land is either used for agriculture or for housing. Following the traditional monocentric model after Alonso et al. (1964), Muth (1969), and Mills (1967), urban land use (cities) emerges endogenously due to commuting costs for workers to produce urban goods: urban land is thus more densely populated than rural land and the urban fringe corresponds to the longest commute of a worker producing urban goods. Importantly, the rental price of land at the fringe of the city must be equalized across potential usages—the marginal productivity of land in the rural sector (agriculture) determining the opportunity cost of expanding further urban land. The last important components of our theory are the drivers of structural change. Structural change is driven by the combination of non-homothetic preferences on the demand side, particularly a subsistence consumption for the rural good, and increasing (agricultural)
productivity on the supply side. This generates transitory dynamics with rising productivity in agriculture that are at the heart of our story: in the old times, due to low agricultural productivity, land is scarce with high values of farmland with respect to income. Moreover, households devote a large fraction of their resources to feed themselves and cannot afford large homes. Few urban workers are concentrated on a very small area and urban land is very densely populated. Later on, with agricultural development, farmland is getting less valuable. This frees up rural land for cities to expand, accommodating rising demand for housing of more numerous urban workers. The city sprawls and average urban density falls through two channels: the fall in the rental price of farmland at the urban fringe and the increasing share of spending towards housing. Note that the decline in urban density occurs even without improvements in the commuting technology—the usual source of sprawling in urban economics. At the latest stages of the transition, in more recent times, the reallocation of workers and land use slows down. Urban expansion slows, urban density declines less and land prices increase at faster rate. As a side-product, we also show how commuting frictions together with location-specific land values generate a wedge between the workers marginal productivities in the rural and urban sector, an ‘agricultural productivity gap’ (Gollin et al. (2014)).

The other natural candidate to account for urban sprawl over time is the development of faster urban commutes, which made urban households live further away from work. Building upon LeRoy and Sonstelie (1983) and DeSalvo and Huq (1996), we incorporate into our theory a commuting mode choice model, which allows for an endogenous decision of individuals of how to commute, based on their opportunity cost of time and location. More specifically, as the opportunity cost of time in the city increases with rising urban productivity, workers optimally choose faster commuting modes and live further away from the center: the city expands at the expense of rural land.¹ Thus, although the mechanisms are entirely different, both urban and rural productivity growth lead to sprawling and suburbanization together with a decline in average urban density. However, the implications for density across urban locations are different. Increasing urban productivity and faster commutes lead to a reallocation of urban workers away from the center towards the city fringe. As a consequence, central density falls more than average urban density since suburban density increases. To the contrary, increasing agricultural productivity and structural change lead to the addition of lower and lower density settlements at the fringe of cities: suburban density falls more than the average urban density. While central density did fall since the mid-nineteenth century, historical data for Paris shows that it fell less than the average urban density. This suggests that both channels—the structural change and the commuting speed channels—have been playing a role in driving the density decline.

Regarding the price of land, we also show that agricultural productivity growth and structural change are crucial to understand their evolution. If land reallocation away from agriculture towards urban use was only driven by urban productivity growth and faster commutes, rural land would be getting scarcer and more valuable: the value of farmland and agricultural rents (i.e. rental income)

¹In our theory, commuting costs (as a share of income) falls endogenously as individuals choose faster commuting modes when urban income increases. Results are qualitatively if one assumes an exogenous fall in commuting costs.
would increase, as a share of income. Agricultural land rents would also become relatively more important than urban ones – predictions that are widely counterfactual to the evidence in Piketty and Zucman (2014). Quite differently, structural change driven by increasing rural productivity frees up farmland, lowering its value relative to income and reducing the importance of agricultural land rents to the profit of urban ones. These predictions are much more in line with the data.

In a third contribution, we develop a quantitative version of our spatial equilibrium model applied to the French context since 1840. Using data from various historical sources, we measure sectoral factors of production and productivities over long period and calibrate our model to fit the process of structural change in France. We show that the quantitative predictions of the model match relatively well the joint evolution of population density and land values over time and across space. We also disentangle the relative importance of falling commuting costs relative to our novel mechanisms based on structural change in explaining the persistent decline in urban density—emphasizing further the quantitative importance of improvements in agricultural productivity for the expansion of cities.

Related literature. The paper relates to several strands of literature in macroeconomics and spatial economics. From a macro perspective, it relates to the literature linking productivity changes and land values, starting with Ricardo (1817). This traditional view would imply that a fixed factor such as land should continuously rise in value with economic development (see, among others, Nichols (1970) and Grossman and Steger (2017) for a recent contribution). However, such a prediction would not fit well the measurement of housing prices and land values over a long period as in Piketty and Zucman (2014) and Knoll et al. (2017) (see also Davis and Heathcote (2007) for related U.S. evidence). An alternative view developed in Miles and Sefton (2020) argues that the rise in land and housing prices can be mitigated by improvements in commuting technologies, which allow cities to expand outwards. Our approach, in the tradition of the theory of structural change, also argues that land used to be scarce and valuable with low productivity in agriculture but rising productivity alleviates pressure on land—putting downward pressure on its value. In a sense, our theory reconciles these different views in a unified framework. From a theoretical perspective, we contribute to the literature on structural change, surveyed in Herrendorf et al. (2014), by considering a spatial dimension—adding an endogenous use of land and a housing sector—in the most conventional multi-sector model with non-homothetic preferences (Kongsamut et al. (2001), Gollin et al. (2007), Herrendorf et al. (2013), Boppart (2014), Comin et al. (2021)). Structural change and urbanization are known to be tightly linked (Lewis (1954)). Gollin et al. (2016) shows that not only economic development but also natural resources rents lead to urbanization. However, the literature has rarely investigated the spatial dimension of structural change, largely abstracting from spatial frictions. Michaels et al. (2012) and Eckert et al. (2018) are notable exceptions. The crucial difference to those is the ability of our framework to replicate the evolution of population density within locations—putting emphasis on the internal structure and density of cities—, while their focus is more on the distribution of population and the sectoral specialization across regions. We also emphasize the implications for land values across time and space, largely absent in these
studies. Adding a spatial dimension to a multi-sector model of structural change also generates endogenously an ‘agricultural productivity gap’ (Gollin et al. (2014)) due to the mere presence of commuting frictions and location-specific housing. This provides a complementary explanation to urban-rural wage gaps, different from migration costs or selection of migrants towards cities (Restuccia et al. (2008), Lagakos and Waugh (2013), Young (2013)).

Our paper also contributes to the literature in spatial economics on urban expansion surveyed in Duranton and Puga (2014, 2015). An important feature of our framework is the existence of preferential residential locations within cities, shaping the population density across space, due to the presence of commuting frictions (Alonso et al. (1964); Muth (1969); Mills (1967)). We expand this literature by bringing the endogenous sectoral allocation of factors and the general equilibrium structure at the heart of the macro literature. Importantly, contrary to the bare bone urban monocentric model, land is in fixed supply and the price of land at the boundary of the city becomes an endogenous object itself affected by the process of structural change. The most related work to our approach developed in Brueckner (1990) shows how location-specific land values pin down rural-urban migrations and the extent of urbanization in a spatial equilibrium (see also Brueckner and Lall (2015) for a survey). However, without the drivers of structural change and endogenous land values at the urban fringe as in our framework, this approach stays relatively silent regarding the long-run dynamics of urbanization and land values. In this latter dimension, our work relates to the literature measuring and explaining land values across space (see Glaeser et al. (2005), Alouy (2016), Alouy et al. (2018) and Combes et al. (2018) for recent contributions). In particular, we show that the dispersion of land values across space and the scarcity of land in some locations depend very much on the extent of economic development and structural change. Our approach also provides an alternative mechanism generating a large sprawling of cities together with economic development. More specifically, it explains, why, over time, most cities expand faster in area than in population as documented on a global scale by Angel et al. (2010). In the French context, we also relate to the historical measurement of urban land use in Combes et al. (2021). Our story is complementary to the usual explanations based on the improvement of commuting technologies and/or the relocation of economic activity within cities (see references in Glaeser and Kahn (2004) and Heblich et al. (2018), Redding (2021) for recent contributions). Lastly, our paper contributes to the literature on quantitative spatial economics surveyed in Redding and Rossi-Hansberg (2017) (see also Ahlfeldt et al. (2015)) by emphasizing the extensive margin of cities.

The paper is organized as follows. Section 2 provides motivating empirical evidence on land use, land values, urban expansion and population density across space over long period in France. Section 3 provides a baseline spatial general equilibrium model of land use and structural change which enlightens the main mechanisms. Section 4 develops a quantitative version calibrated to French historical data. Section 5 concludes.
2 Historical Evidence from France

2.1 Land use and Employment in Agriculture

Data. Using various sources described in Appendix B\textsuperscript{2}, we assemble aggregate data on employment shares in agriculture and agricultural land use since 1840. Historical data on land use in agriculture are available roughly every 30 years (or less) until the 1980s and then at higher frequency. They are largely extracted from secondary sources based on the Agricultural Census (Recensement Agricole), and cross-checked with various alternative historical sources (Toutain (1993) among others). Post-1950, data are from the Ministry of Agriculture.

Employment. As all countries going through structural transformation, France exhibits a large reallocation of labor away from agriculture over the period, from about 60\% employed in agriculture in 1840 to about 2.5\% today (Figure 1, dashed line).\textsuperscript{3} The process of structural change accelerated significantly over the period 1945-1975: in 1945, 36\% of the working population are still in agriculture and this number falls below 10\% in 1975. In this sense, France is a bit peculiar relative to the other advanced economies: it is still a very agrarian economy right after World War II—much more than the U.K. or the U.S. This measurement is described in detail in Appendix B.2.

Land use. Although measurement is sometimes difficult for the very early periods, one can confidently argue that, in the aggregate, the share of French land used for agriculture significantly fell since 1840 (Figure 1, solid line).\textsuperscript{4} Our preferred estimates are that about two thirds of French land was used for agriculture in 1840. In 2015, this number is down to 52\%. In other words, about 15\% of French land use has been reallocated away from agriculture. While this might not seem like a very large number, this is very large from the perspective of urban expansion. 15\% of the French territory is actually more than the total amount of land with artificial use in France nowadays (about 9\% of total land today).\textsuperscript{5} While this is difficult to assess over such a long period, the novel usage of the land formerly used in agriculture, it is likely that a significant fraction of this land has been artificialized—allowing cities to expand. More precise data on land use over the period 1982-2015 show that the surface of artificialized soils increased by about 2 millions of hectares (3.7\% of the French territory), about 70\% of the quantity of the land no longer used for agriculture over the same period.\textsuperscript{6} The measurement of cities area (presented below) provides further compelling evidence that a significant fraction of agricultural land was reallocated towards urban land use. We

\textsuperscript{2}The appendix is available at https://floswald.github.io/pdf/landuse-appendix.pdf

\textsuperscript{3}Estimates of rural population are also available for the same time-period (see Appendix B.2). Rural population follows a similar path with, as expected, higher levels as many people in rural areas do not work directly in agriculture. One needs to be cautious though when using data on rural vs. urban population as the (ad-hoc) definition by official statistics varies over the period.

\textsuperscript{4}The main issue is the definition of agricultural land (in particular, the allocation of grazing fields) which is not entirely consistent across years before World War II. See Appendix B.1 for details.

\textsuperscript{5}Since 1982, data on land use beyond agricultural land use are available on a regular basis from the Enquetes Teruti and Teruti-Lucas.

\textsuperscript{6}The rest of agricultural land is to a large extent converted into forests and woods (Enquetes Teruti and Teruti-Lucas). Their surface, including groves and hedges, increased by almost 1 million of hectares between 1982 and 2015.
Figure 1: Land use and labor reallocation in France (1840-2015).

Notes: The solid line shows the share of French land used for agriculture (left-axis). The dashed line shows the share of workers in the agricultural sector (right-axis). Source: See Appendix B.1.

present details in Appendix B.1.

2.2 Urban Expansion

Data. We use maps, aerial photographs and satellite data to measure the area of the main French cities at different dates: 1866 (military maps, e.g. carte d’Etat Major), 1950 (maps and/or photographs), and every ten to fifteen years after 1975 using satellite data from the Global Human Settlement Layer (GHSL) project. One caveat of our area measurement is that we cannot have any measurement between 1866 and 1950. Data and procedure for the measurement of urban extent across French cities are detailed in Appendix B.7. Measurement of the urban extent using maps in 1866 and 1950 is performed for the 100 largest cities in population in the initial period. For a given city, the urban extent ends when the land is not continuously built. For the satellite data, it is delimited by grid cells where the fraction built is below 30% and a requirement that cells are connected. By way of example, Figures VII and VIII in Appendix B.7.1 show the area measurement for a medium-size French city, Reims, in 1866 and 1950 using maps. Figure XVI shows the

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7We double-check the quality of photo/map measurement in the most recent period relative to satellite data measurement (see Appendix B.7.5). The cross-sectional correlation between both measures is very high. We also cross check our measures with Angel et al. (2010) for Paris and find very similar results.

8Measurement is not very sensitive to alternative thresholds (see Appendix B.7.6). Figures XII and XIII in the same Appendix illustrate how GHSL data are used to delineate the urban boundaries of Marseille and Bordeaux.
same city of Reims in 2016 viewed from the sky, with an area of about 57 km$^2$—about 20 times larger than its 1866 counterpart. This last figure also clearly shows how the city is surrounded by agricultural land—a crucial element for our story where urban land expands out of farmland. This feature is not specific to Reims. Recent satellite observations from the Corine Land Cover project—further detailed in Appendix B.8—show that our sample of cities are surrounded mainly by agricultural land: apart from their coastal part and water bodies, two thirds of land use in the near surroundings of cities is agricultural.\footnote{The rest is made of forest/moors and discontinuous urban land (e.g. leisure/transport infrastructure, industrial/commercial sites, ...)—both categories in roughly equal proportions. See details in Appendix B.8.}

Using Census data, we relate the measured land area used by cities to the corresponding population. Data for the first available Census in 1876 are used for the initial period of study. Census data defines population at the municipality level (‘commune’) and an urban area can incorporate more than one municipality. In 1870, this is not a major concern as the main ‘commune’ of the city is the whole city population. In later periods, one needs to group municipalities (‘communes’) into an urban area. Post 1975, GHSL data combines satellite images with Census data on population. This directly provides the population of every grid cells of our measured urban area, circumventing the issue. However, for the 1950 period, the different municipalities that are part of our measured areas must be selected. This is done on a case by case basis, looking at the map of each of the 100 largest urban areas. This way, we make sure that the overall population of the area incorporates all the corresponding municipalities’ population. The procedure is detailed in Appendix B.7.2.\footnote{For most cities in 1950, only very few ‘communes’ are agglomerated into one city. Only the largest cities, and particularly Paris, are the results of the agglomeration of many different ‘communes’.}

**The area and population of French cities.** Not surprisingly, more populated cities are larger in area at a given date. However, in the cross-section, the urban area increases strictly less than one for one with urban population: more populated cities are denser on average.\footnote{In the cross-section, at a given date, a 10% increase in the population of a city corresponds to approximately a 8.5% increase in its area and this elasticity varies fairly little across the different time periods.} This stands in contrast with their evolution in the time-series. Over time, cities have been increasing much faster in area than in population. Let us give some order of magnitude and describe the average evolution over time for the most populated 100 French cities in 1876. Figure 2 shows the evolution of the total area and population of these 100 cities over the period considered—both variables being normalized to 1 to show the increase in size. Since 1870, the area of cities has been multiplied by a factor close to 30 on average. This is very large. Between 1870 and 1950, the area of cities was roughly multiply by a factor 6. Between 1950 and today, the area of cities was multiplied again by a factor 5 on average—the fastest rate of increase being observed over the period 1950-1975. For comparison, the population of these largest cities has been multiplied by a factor close to 4 since 1870.\footnote{French population was multiplied by a bit less than 2 over the entire period. Due to the reallocation of people way from rural areas towards cities, we get roughly a factor 4 over the period.} As urban area increased at a much faster rate than urban population, the average urban density significantly declined over the period.
Figure 2: Urban area and population of the 100 largest cities in France (1870-2015).

Notes: The dashed line shows the total urban area of the 100 cities relative to the initial period (sum of all the urban areas). The bottom solid line shows the total population relative to the initial period in the same cities. Both area and population are normalized to unity in the initial period. Source: See Appendix B.7.

The density of French cities. Using the population and the area of cities at the different dates, one can measure the evolution of urban densities across the different cities over 150 years. While in the cross-section larger cities are denser, the density of French cities declined over time—area expanding at a faster rate than population. This is shown in Figure 3 for the population-weighted average of density across the 100 largest French cities. The average urban density fell massively over the period: density has been divided by a factor of roughly 8. Urban density fell at the fastest rate over the period 1950-1975 and barely falls thereafter. Thus, urban density fell the most over the period when people massively left rural areas and the employment share in agriculture also fell the most. The later slowdown of the decline in density coincides with the slowdown in the rate of structural transformation.\footnote{The historical decline in urban density is observed across all cities although the magnitude differs across cities. See Appendix B.7.4 for further insights on the evolution of urban density across different cities.}

Ideally, one would like to explore how density evolved in different locations of a city (within-city variations). This would provide information on whether density fell in the central locations or in the outskirts of the city. Unfortunately, for most cities we are not able to differentiate the central density to the suburban one as most cities expand the area of their main historical ‘commune’, particularly so over the period 1870-1950. Thus, we cannot measure the historical population in different parts of a city. However, it can be done for Paris which is divided into several districts.
Figure 3: The historical decline in urban density.
Notes: The solid line shows the urban density averaged across the top 100 French cities (weighted average with 1975 population weights). Source: Etat major, IGN, GHSL and Census. See Appendix B.7 for details.

Figure 4: The historical decline in urban and central density in Paris.
Notes: The solid line shows the average urban density in Paris; the dashed line shows the density in Central Paris (districts 1 to 6). Source: Etat major, IGN, GHSL and Census.
Figure 4 shows the evolution of the density of Central Paris relative to the average urban density of the metropolitan area: the central density of Paris did fall over time but significantly less than the average density of the city. This suggests that the decline in average urban density is not only due to a reallocation of urban residents away from dense centers but also due to the addition of less and less dense suburban areas at the city fringe over time.

2.3 Land values

Data. Data on land and housing values (over income) for France over a long period can be found in Piketty and Zucman (2014). Historical data for the real housing price index for France are provided in Knoll et al. (2017).

Historical evolution. Figure 5a describes the evolution of the aggregate value of French land over income since 1820. The fall in the value of housing and land wealth (as a share of income) in the pre-World War II period is essentially driven by a declining value of farmland. While farmland was expensive relative to income in the nineteenth century, it is today relatively cheap. This is confirmed by data on average farmland prices: since 1850, the average value of an agricultural field (per unit of land) as a share of per capita income has been divided by a factor of 15 in France. This fact is at the heart of our story: structural change puts downward pressure on farmland values—allowing cities to expand at a fast rate. As a consequence, there is an important reallocation of

14Using various data sources, we also computed a measure of farmland prices per unit of land. Our estimates are consistent with Piketty and Zucman (2014).
land values across usage, from agricultural land towards housing (or urban) land. While the value of agricultural land accounted for more than 70% of housing and land wealth in 1820, it accounts for only 3% in 2010. Lastly, despite the falling value of farmland as share of income, the total value of housing and land wealth (as share of income) grows at an increasing rate after 1950.

This steep increase, arguably driven by the increasing value of urban land where most of the population is concentrated, echoes the findings of Knoll et al. (2017). They show that for developed countries, including France, housing prices have been quite stable until the 1950s before rising at an increasing pace—a hockey-stick shape of housing prices as shown in Figure 5b.

To sum-up, our historical data shows a set of salient facts over the last 180 years: beyond the well-known reallocation of labor away from agriculture, land has been reallocated away from agricultural use. Migrations away from the rural areas were accompanied with urban expansion both in area and population. However, given that urban area grew at a significantly faster pace than urban population, the average urban density massively declined over the period, particularly so in the decades following World War II in France. Together with this process of structural change, the value of farmland as a share of income shrank a lot to the benefits of non-agricultural (urban) land. These stylized facts motivate our subsequent theoretical analysis where we introduce a spatial dimension together with endogenous land use to the most standard theory of structural change with non-homothetic preferences.

3 A Baseline Model

3.1 Production

We consider an economy producing an urban good \( u \) and a rural good \( r \). The urban good is thought as a composite of manufacturing goods and services, while the rural good is thought as an agricultural good. The urban good is also used in the production of housing services. Goods and factor markets are perfectly competitive. Both goods are perfectly tradable.

Factor Endowments. The economy is endowed with land and a continuum of workers, both in fixed supply. Land area is denoted \( S \). Land can be used to produce the rural good or for residential purposes. Each worker is endowed with one unit of labor and we denote by \( L \) the total population of workers.

Production and Factor Payments. The production of the urban good only uses labor as input. One unit of labor produces \( \theta_u \) units of the urban good. Perfect competition insures that the urban wage is

\[
w_u = \theta_u,
\]

\(1\)

Bonnet et al. (2019) show that this increase in the price of housing is largely driven by the price of land and not by the capital and structure component.
in terms of units of the urban good, which is used as numeraire.\textsuperscript{16} Aggregate production of the urban good is

\[ Y_u = \theta_u L_u, \]

where \( L_u \) denotes the number of workers working in the urban sector.

The production of the rural good uses labor and land according to the following constant returns to scale technology

\[ Y_r = \theta_r (L_r)^{\alpha} (S_r)^{1-\alpha}, \]

where \( L_r \) denotes the number of workers working in the rural (agricultural) sector, \( S_r \) the amount of land used for production and \( \theta_r \) a Hicks-neutral productivity parameter. \( 0 < \alpha < 1 \) is the intensity of labor use in production, \( 1 - \alpha > 0 \) ensures that land is used more intensively to produce the rural good.\textsuperscript{17}

Define \( p \) the relative price of the rural good in terms of the numeraire urban good. Rural workers and land are paid their marginal productivities,

\[ w_r = \alpha p \theta_r \left( \frac{S_r}{L_r} \right)^{1-\alpha}, \quad (2) \]

\[ \rho_r = (1 - \alpha) p \theta_r \left( \frac{L_r}{S_r} \right)^{\alpha}, \quad (3) \]

where \( w_r \) is the rural wage and \( \rho_r \) the rental price of land anywhere in the rural sector.

Remark. The important technology assumption is that the rural sector uses a fixed factor, land, for production, which implies (stronger) decreasing returns to scale to labor in this sector compared to the urban sector. The fact that the urban sector does not use land is not crucial as long as this sector is less land intensive than the rural one.

### 3.2 Spatial Structure and Commuting Costs

**Spatial structure.** Total available land \( S \) is devoted to either housing or rural goods production. The production of the urban good takes place in the city, while the production of the rural good, being more land intensive, takes place in the rural area. For now, we assume that production of the urban good takes place in only one location \( \ell = 0 \). An extension with multiple locations (multiple cities) is provided in Appendix C.2.

One can think of \( \ell = 0 \) as the Central Business District (CBD) in a standard urban model, where space is given by the interval \([0, S]\). Workers’ residence \( \ell \) can lie anywhere in this interval, however, they face spatial frictions \( \tau(\ell) \) when commuting to work in the urban sector. A worker residing in location \( \ell \) and working in the urban sector earns wage *net of spatial frictions* equal to \( w(\ell) = \)

\textsuperscript{16}For now, we consider the urban productivity \( \theta_u \) (and thus the urban wage) as exogenous. We consider agglomeration forces in Section 4.

\textsuperscript{17}The model is extended in Appendix C.1.1 to explore a more general CES production function for the rural good.
$w_u - \tau(\ell)$, with $\tau(0) = 0$ and $\partial \tau(\ell)/\partial \ell \geq 0$. The commuting cost $\tau(\ell)$ incorporates all spatial frictions which lower disposable income available for consumption when living further away from the location of production. It includes time-costs of commuting and the effective spending on transportation.\textsuperscript{18} The commuting cost is partly endogenous in our framework, because urban households adjust their mode of commuting depending on their income and their location, as described in details below.

Since spatial frictions increase with $\ell$, urban workers locate as close as possible to $\ell = 0$. If one denotes $\ell = \phi < S$ the furthest away location of an urban worker, $\phi$ is endogenous in our framework and represents the fringe of the city. Workers residing in locations beyond $\phi$ produce the rural good, which does not involve spatial frictions, as rural workers do not commute.

\textit{Remarks.} The spatial structure calls for a number of important remarks. First, if it were possible for all workers to locate at $\ell = 0$, there would be no spatial frictions. Second, one should note that for $\ell \leq \phi$, land will be used for residential purposes to host urban workers. As a consequence, land available for rural production would also be maximized if all workers could locate at $\ell = 0$. This case could correspond to an entirely ‘vertical’ city, where land use and spatial frictions are irrelevant. We view this extreme case as a standard two-sector model of structural transformation. Last, the spatial frictions $\tau(\ell)$ do not involve traffic congestion in the baseline—the reason why a more compact city (lower $\phi$) always saves on commuting costs in our baseline economy. We allow for congestion and agglomeration effects in Appendix D.4.

\textbf{Commuting costs.} We provide a micro-foundation for the commuting costs, $\tau(\ell)$, where urban workers choose a commuting mode $m$ depending on their location $\ell$ and opportunity cost of time (wage rate $w_u$). This modelling approach helps mapping commuting costs into observables from commuting data but results do not depend qualitatively on the micro-foundation as long as commuting costs are increasing in the opportunity cost of time and commuting distance.

Commuting costs in location $\ell$, $\tau(\ell)$, are the sum of spending on commuting using transport mode $m$, $f(m)$, and time-costs proportional to $w_u \cdot t(\ell)$, where $t(\ell)$ denotes the time spent on daily commutes of an individual located in $\ell$, such that

$$\tau(\ell) = f(m) + \zeta w_u \cdot t(\ell), \quad (4)$$

whereby $0 < \zeta \leq 1$ represents the valuation of commuting time in terms of foregone wages. Transportation modes $m$ available are optimally chosen. They are continuously ordered by their speed, as in DeSalvo and Huq (1996), such that $m$ denotes both the mode and the speed of commute. Faster commutes are more expensive and $f(m)$ is increasing in $m$. For tractability, we use the following functional form, $f(m) = \frac{c_\tau m^{\eta_m}}{\eta_m}$, with $\eta_m > 0$ and $c_\tau$ a cost parameter measuring the efficiency of the commuting technology. With speed $m$, the commuting time (both ways) is equal to $\frac{2 \ell}{m}$. This

\textsuperscript{18}It could also incorporate an income reduction if it is harder to find a job when living further away from the location of production.
yields the following expression for the commuting costs,

$$\tau(\ell) = \frac{c_{\tau}}{\eta_m} m^{\eta_m} + 2\zeta w_u \left(\frac{\ell}{m}\right).$$

(5)

This expression of the commuting costs facilitates parametrization and preserves some tractability, while elucidating the main mechanisms.\(^\text{19}\) We now turn to the optimal choice of transportation mode.

**Optimal mode of transportation.** At any given moment in time, prevailing technology offers different transportation modes ordered by their respective speed \(m\). An individual in location \(\ell\) chooses the mode of transportation corresponding to speed \(m\) in order to minimize the commuting costs \(\tau(\ell)\). By equalizing the marginal cost of a higher speed \(m\) to its marginal benefits in terms foregone wage, the optimal chosen mode/speed satisfies,

$$m = \left(\frac{2\zeta w_u}{c_{\tau}}\right)^{1-\xi} \cdot \ell^{1-\xi},$$

(6)

where \(\xi \equiv \frac{\eta_m}{1+\eta_m} \in (0,1]\). Individuals living further away choose faster commuting modes. The speed of commuting also increases with the wage rate as a higher wage increases the opportunity cost of time. Using Equations 5-6, we get that equilibrium commuting costs satisfy,

$$\tau(\ell) = a \cdot (w_u \ell)^{\xi},$$

(7)

where \(a \equiv \left(\frac{1+\eta_m}{\eta_m}\right) c_{\tau}^{1+\eta_m} (2\zeta)^{\eta_m} > 0\). Commuting costs are falling with improvements in the commuting technology (lower \(a\)).\(^\text{20}\) They are increasing with the wage rate (the opportunity cost of time) and the commuting distance with constant elasticities. Since individuals optimally choose the commuting speed, the elasticity \(\xi\) of commuting cost to the wage rate is strictly smaller than unity. This is important as it implies that, for a given residential location, the share of resources devoted to commuting falls with rising urban productivity and wages. In equilibrium, this tends to make individuals willing to live further away when productivity increases in order to enjoy larger homes. Lastly, our derivation of commuting costs enlightens the calibration as the elasticity of commuting costs to commuting distance (resp. income) is directly tied to the elasticity of commuting speed to commuting distance (resp. income), which have data counterparts (Equation (6)).

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\(^{19}\)The cost \(f(m)\) has several possible interpretations. At a more macro level, it can represent the fixed cost of installing public transportation, where a faster mode is more expensive (a train line versus the horse drawn omnibus). At a more individual level, it represents the cost of buying an individual mean of transportation—a bike being cheaper than an automobile. However, this reduced-form approach sets aside the possibility that the implemented commuting technologies and the effective speed of commuting depends in a more sophisticated way on the equilibrium allocation in the city (e.g. traffic congestion or the construction of transportation infrastructures may depend on the whole spatial allocation of urban residents). The extension in Appendix C.1.2 uses a more general function for the spending on commuting \(f\), also increasing in the commuting distance \(\ell\) and urban wages \(w_u\): \(f = f(m,\ell, w_u)\). Longer commutes are more expensive and higher urban labor costs also increase commuting costs.

\(^{20}\)\(a\) is alike a relative price of commuting: if technology improves relatively faster in the commuting sector, the relative price \(a\) of commuting (in terms of urban goods) falls.
3.3 Preferences and Consumption

Preferences. Preferences over urban and rural goods are non-homothetic as in Kongsamut et al. (2001) and Herrendorf et al. (2013) among others. Consider a worker living in a location $\ell$. Denote $c_r(\ell)$ the consumption of rural (agricultural) goods, $c_u(\ell)$ the consumption of urban goods (used as a numeraire) and $h(\ell)$ the consumption of housing. The composite consumption good is

$$C(\ell) = (c_r(\ell) - \xi)^{\nu(1-\gamma)}(c_u(\ell) + \xi)^{(1-\nu)(1-\gamma)}h(\ell)^{\gamma}$$ (8)

where $\xi$ denotes the minimum consumption level for the rural good, and where $\xi$ stands for the initial endowment of the urban good. Preference parameters $\nu$ and $\gamma$ belong to $(0,1)$. Workers derive utility only from consumption. The utility of a household in location $\ell$ is equivalent to $C(\ell)$.

Budget constraint. The household earns a wage income net of spatial frictions $w(\ell)$ in location $\ell$. Given the spatial structure, $w(\ell) = w_u - \tau(\ell)$ for $\ell \leq \phi$ and $w(\ell) = w_r$ for $\ell > \phi$. The households also earns land rents, $r$. Land rents are redistributed lump-sum equally across workers and are thus assumed to be independent of location. The budget constraint of a worker in location $\ell$ satisfies

$$pc_r(\ell) + c_u(\ell) + q(\ell)h(\ell) = w(\ell) + r,$$ (9)

with $q(\ell)$ the rental price per unit of housing (henceforth the housing price) in location $\ell$.

Expenditures. Maximizing utility (Equation (8)) subject to the budget constraint (Equation (9)), expenditures on each yields

$$pc_r(\ell) = (1-\gamma)\nu(w(\ell) + r + \xi - pc) + pc$$ (10)

$$c_u(\ell) = (1-\gamma)(1-\nu)(w(\ell) + r + \xi - pc) - \xi$$ (11)

$$q(\ell)h(\ell) = \gamma(w(\ell) + r + \xi - pc).$$ (12)

Due to the presence of subsistence needs ($\xi > 0$), individuals reallocate consumption away from the rural good with rising income, increasing the consumption share of the urban good and housing. The reallocation of demand towards the urban good is stronger when $\xi > 0$.

3.4 Equilibrium Sorting

Mobility equations. We consider an equilibrium, where ex-ante identical workers sort across locations. Since the rural and the urban goods are perfectly tradable, urban workers, which would all prefer locations closer to $\ell = 0$, compete for these locations. Adjustment of housing prices through the price of land makes sure that households remain indifferent across different locations. Using Equations (10)-(12), this implies the following mobility Equation, where consumption is
equalized to $C$ across locations $\ell$,

$$C = C(\ell) = \kappa \frac{w(\ell) + r + s - pc}{q(\ell)^\gamma},$$

(13)

with $\kappa$ constant across locations, equal to $((1 - \gamma)\nu(1-\gamma)/(1-\gamma)(1-\nu))^\gamma/\nu(1-\gamma)$.

The mobility Equation (13) implies that $((w(\ell) + r + s - pc)/q(\ell)^\gamma)$ is constant across locations. This holds within urban locations ($\ell \leq \phi$), within (identical) rural locations as well as when comparing an urban and rural worker. Since workers in the rural sector do not face spatial frictions and live in ex-post identical locations, $\ell \geq \phi$, the price of housing must be the same across these locations. We denote by $q_r$ the price of housing in the rural sector, where $q_r = q(\ell \geq \phi)$. A worker in the rural sector is paid his marginal productivity $w_r$, receives land rents $r$ and faces the same housing price $q_r = q(\phi)$ than an urban worker at the fringe. Therefore we have

$$w(\phi) = w_r = w_u - \tau(\phi).$$

(14)

In other words, the urban worker at the urban fringe must have the same wage net of commuting frictions than a rural worker. Equation (14) is essential to understand the spatial allocation of workers: higher spatial frictions at the fringe $\phi$ reduce incentives of rural households to move to the city. Equation (14) also shows how the spatial structure matters to understand the urban-rural wage gap.

**Housing Rental Price Gradient.** Within city locations ($\ell \leq \phi$), the housing price adjusts such that workers are indifferent across locations. Using Equations (13) and (14), we get

$$q(\ell) = q_r \left( \frac{w(\ell) + r + s - pc}{w(\phi) + r + s - pc} \right)^{1/\gamma} = q_r \left( \frac{w(\ell) + r + s - pc}{w_r + r + s - pc} \right)^{1/\gamma}. $$

(15)

Within the city, $q(\ell)$ is falling with $\ell$ to compensate workers who live in worse locations. For $\ell$ above $\phi$, the housing price is constant across locations and equal to $q_r$. A crucial difference compared to the standard urban model is that the price at the fringe $q_r$ is endogenously determined in our general equilibrium model.

**3.5 Housing Market Equilibrium**

**Housing Demand.** Using Equation (15), the demand for housing space per worker in each location $h(\ell)$ is increasing with $\ell$ for $\ell \leq \phi$,

$$h(\ell) = \gamma \left( \frac{w(\ell) + r + s - pc}{q(\ell)} \right) = \left( \frac{\gamma}{q_r} \right) (w(\phi) + r + s - pc)^{1/\gamma}(w(\ell) + r + s - pc)^{1-1/\gamma}. $$

(16)

Facing higher housing prices, household closer to the CBD demand less housing space. For locations in the rural area, housing demand per rural worker is constant equal to $h(\ell \geq \phi) = \gamma \left( \frac{w_r + r + s - pc}{q_r} \right)$. 

18
Housing Supply. The supply of housing (floorspace) is provided by land developers, which can use more or less intensively the land for residential purposes. In each location $\ell$, developers supply housing space $H(\ell)$ per unit of land with a convex cost, $\frac{H(\ell)^{1+1/\epsilon}}{1+1/\epsilon}$ with $\epsilon > 0$, paid in units of the numeraire.\(^{21}\) Profits per unit of land of the developers are

$$\pi(\ell) = q(\ell)H(\ell) - \frac{H(\ell)^{1+1/\epsilon}}{1+1/\epsilon} - \rho(\ell),$$

where $\rho(\ell)$ is the rental price of a unit of land in location $\ell$ (henceforth the land price). Similarly to the housing price $q(\ell)$ above, for locations beyond the fringe $\phi$, the land price is constant, hence $\rho_r = \rho(\ell \geq \phi)$.

Maximizing profits gives the following supply of housing $H(\ell)$ in a given location $\ell$,

$$H(\ell) = q(\ell)^\epsilon, \quad (17)$$

where the parameter $\epsilon$ is the price elasticity of housing supply. More convex costs to build intensively on a given plot of land reduces the supply response of housing to prices.\(^{22}\) Lastly, free entry imply zero profits of land developers. This pins down land prices in a given location,

$$\rho(\ell) = \frac{q(\ell)H(\ell)}{1+\epsilon} = \frac{q(\ell)^{1+\epsilon}}{1+\epsilon}. \quad (18)$$

Equation (18), together with Equation (15), implies that land prices are higher in locations closer to the city center, more so if land developers can build more intensively (higher $\epsilon$).

Arbitrage across land use implies that the land price in the urban sector, $\rho(\ell)$, must in equilibrium be above the marginal productivity of land for production of the rural good (Equation (3)), where the condition holds with equality in the rural part of the economy, for $\ell \geq \phi$,

$$\rho_r = \frac{q_r^{1+\epsilon}}{1+\epsilon} = (1-\alpha)p\theta_r \left(\frac{L_r}{S_r}\right)^\alpha. \quad (19)$$

Importantly, this equation shows that a fall in the relative price of rural goods and/or a reallocation of workers away from the rural sector lowers the price of urban land at the city fringe.

Housing Market Clearing. Consider first locations within the city, $\ell \leq \phi$. Market clearing for housing in each location implies $H(\ell) = D(\ell)h(\ell)$, where $D(\ell)$ denotes the density (number of urban workers) in location $\ell$. Within the city, the density $D(\ell)$ follows from Equations (16) and (17), hence

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\(^{21}\) The urban good is used as an intermediary input for the production of housing space. $1/\epsilon > 0$ is a cost parameter measuring the convexity of the cost function. In extension C.1.3, we use a more general cost function. The parameter $\epsilon = \epsilon(\ell)$ can depend on the location.

\(^{22}\) Some equivalent formulation holds for a Cobb-Douglas production function of housing (see Combes et al. (2018)).
\[ D(\ell) = \frac{H(\ell)}{h(\ell)} = \frac{q(\ell)^{1+\epsilon}}{\gamma(w(\ell) + r + s - p_c)}. \]  

Density for \( \ell \leq \phi \) can be rewritten using Equation (15) and Equation (18) as

\[ D(\ell) = \rho_r \frac{1 + \epsilon}{\gamma} (w(\phi) + r + s - p_c)^{-\frac{1+\epsilon}{\gamma}} (w(\ell) + r + s - p_c)^{\frac{1+\epsilon}{\gamma} - 1}. \]

Importantly, a lower rural land price \( \rho_r \) at the urban fringe lowers density across all urban locations. Integrating density defined in Equation (21) across urban locations gives the total urban population,

\[ L_u = \int_0^\phi D(\ell) d\ell = \rho_r \int_0^\phi \frac{1 + \epsilon}{\gamma} (w(\phi) + r + s - p_c)^{-\frac{1+\epsilon}{\gamma}} (w(\ell) + r + s - p_c)^{\frac{1+\epsilon}{\gamma} - 1} d\ell. \]

Equation (22) pins down the city size \( \phi \). It says that if more workers are willing to move in the urban sector, the city will have to be bigger in area to host them—\( \phi \) is increasing with \( L_u \). One should also notice that the city’s area increases if the price of land \( \rho_r \) at the fringe is lower, if housing supply conditions are tighter (low \( \epsilon \)), and if commuting frictions \( \tau(\ell) \) are lower.

In the rural area, \( \ell \geq \phi \), market clearing for residential housing imposes

\[ L_r \gamma (w_r + r + s - p_c) = S_{hr} (q_r)^{1+\epsilon} = S_{hr} (1 + \epsilon) \rho_r, \]

where \( S_{hr} \) is the amount of land demanded in the rural area for residential purposes. This leads to the following demand of land for residential purposes in the rural area,

\[ S_{hr} = \frac{L_r \gamma (w_r + r + s - p_c)}{(1 + \epsilon) \rho_r}. \]  

**Land and labor market clearing.** Land is used for residential or productive purposes. With total land available in fixed supply \( S \), the land market clearing condition is

\[ S_r + S_{hr} + \phi = S \]

Using Equation (23), this is equivalent to

\[ S_r = S - \phi - \frac{L_r \gamma (w_r + r + s - p_c)}{(1 + \epsilon) \rho_r}. \]

The labor market clearing is such that the total population \( L \) is located either in the city or in the rural area,

\[ L_u + L_r = L. \]  

**Land rents.** Aggregate land rents, \( rL \), include the land rents generated both in the city and in
the rural area,
\[ rL = \int_0^\phi \rho(\ell) d\ell + \rho_r \times (S - \phi), \]
where it is useful to notice that the rental income in the city exceeds the rental income of farmland for the same area due to spatial frictions.

### 3.6 Goods markets equilibrium

A last step consists in clearing the goods market for rural and urban goods to pin down the allocation of labor across sectors for a given equilibrium city size \( \phi \).

**Aggregate per capita income.** Let us introduce \( y \) as the aggregate per capita income in the economy net of spatial frictions that is spent on both goods,
\[ y = r + \frac{L_r}{L} w_r + \frac{1}{L} \int_0^\phi w(\ell) D(\ell) d\ell. \]

**Goods market clearing conditions.** Aggregating Equations (10)-(11) across locations, we get that aggregate per capita consumption of rural good and urban good satisfy
\[ \begin{align*}
pc_r &= \nu (1 - \gamma)(y + \bar{s} - pc) + pc \\
c_u &= (1 - \nu)(1 - \gamma)(y + \bar{s} - pc) - \bar{s}
\end{align*} \]

The rural good is only used for consumption. This gives the following market clearing condition for the rural good,
\[ \nu (1 - \gamma)y + \nu (1 - \gamma)(\bar{s} - pc) + pc = py_r, \]
where \( y_r = \frac{Y_r}{L} \) denotes the production per worker of the rural good.

The urban good market clearing is more involved as urban goods are either consumed, used as intermediary inputs to build residential housing (in all locations) or used to pay for commuting costs. The sum of these three uses equals the supply of the urban good, expressed per capita,
\[ c_u + \frac{1}{L} \int_0^\phi \tau(\ell) D(\ell) d\ell + \frac{1}{L} \frac{\epsilon}{1 + \epsilon} \int_0^S q(\ell) H(\ell) d\ell = y_u, \]
where \( y_u = \frac{Y_u}{L} \) denotes the production per worker of the urban good.

### 3.7 Equilibrium allocation

For a given set of exogenous parameters, technological parameters \((\theta_u, \theta_r, \alpha)\), commuting cost parameters \((a, \xi)\) and resulting spatial frictions \(\tau(\ell)\) at each location \(\ell \in \mathcal{L}\), housing supply conditions \(\epsilon\), and preference parameters, \((\nu, \gamma, \bar{s}, \bar{z})\), the equilibrium is defined as follows:

**Definition 1.** An equilibrium is a sectoral labor allocation \((L_u, L_r)\), a city fringe \(\phi\) and rural land
used for production \((S_r)\), sectoral wages \((w_u, w_r)\), a rental price of farmland \((\rho_r)\), a relative price of rural goods \((p)\) and land rents \((r)\), such that:

- **Factors are paid the marginal productivity**, Equations (1)-(3).
- **Workers are indifferent in their location decisions**, Equation (14).
- **The demand for urban residential land (or the city fringe \(\phi) satisfies Equation (22)**.
- **Land and labor markets clear**, Equations (24) and (25).
- **Land rents satisfy Equation (26)**.
- **Rural and urban goods markets clear**, Equations (27) and (28).

The main intuition for the equilibrium allocation goes as follows: if the urban sector hosts more workers, the area of the city has to be larger \((\phi \text{ tends to increase with } L_u)\). However, if the city is larger in area, the worker in the further away urban location commutes more, making the urban sector less attractive for workers: a higher \(\phi\) reduces the incentives of workers to move from the rural to the urban sector \((L_u \text{ tends to decrease with an increasing } \phi)\). Given technology, the combination of these two forces pins down the allocation of workers across sectors together with the land used for urban residential housing.

However, the equilibrium cannot be described analytically. Thus, we use numerical illustrations to explain the main mechanisms through which increasing productivity, in the rural and urban sectors, change the population, area and density of cities in our framework. The numerical simulations are not aiming at being a measurement tool but at elucidating the main channels at play to understand urban expansion when economies go through the process of structural change. A quantitative evaluation in the context of France is provided in Section 4.

### 3.8 Numerical illustrations

**Parameter values.** We consider an economy as described above endowed with land and labor, both normalized to 1. While the exercise is not quantitative, we nevertheless set parameters values in a reasonable range with respect to the data. The share of land in rural production is set to 25\% \((\alpha = 0.75)\). We set the constant elasticity of housing supply \(\epsilon\) to 4 in the range of empirical estimates. Preferences towards the different goods are set to roughly match the employment share in agriculture and the housing spending share in the recent period in France—\(\nu = 2.5\%\) and \(\gamma = 30\%\). At each date \(t\), the productivity is assumed to be the same in both sectors, \(\theta_{u,t} = \theta_{r,t}\), and the initial productivity is normalized to unity. Both sectors are growing at the same constant rate of productivity growth of 1.25\% per annum. Most importantly, together with rising productivity, structural change emerges due to the presence of subsistence needs for rural goods, \(\xi = 2/3\). As we focus on subsistence needs, we set \(s\) to zero. With such preferences, the share of employment in the rural sector is about 60\% at start. For comparison, we explore at a later stage the model dynamics when structural change is driven by increasing demand for urban goods rather than subsistence.
needs ($g >> c$). The values for the commuting costs parameters are set such that the urban area remains small relative to land used in agriculture, $a = 2$. The parameter determining the elasticity of commuting costs to urban income and commuting distance, $\xi$, is set to $2/3$ to generate an increase in the average urban commuting speed comparable to the data (see Miles and Sefton (2020)).

**Baseline.** Figure 6 summarizes the model dynamics following rising productivity in both sectors—starting at an initial period labeled 1840 for illustration purposes. The top panel shows the evolution of employment, spending shares and relative prices. As well known in the literature, due to low initial (rural) productivity, the share of workers needed to produce rural goods is high at start to satisfy subsistence needs. The demand for rural goods for subsistence makes them initially relatively expensive and households spend a disproportionate share of income on rural goods. With rising (rural) productivity solving the ‘food problem’, workers move away from the rural to the urban sector, the relative price of rural goods falls, as well as the spending share towards rural goods.

The bottom panel of Figure 6 shows outcomes that are more specific to our theory with endogenous land use: urban area (compared to urban population), urban densities (average, central and fringe) and land rents (as a share of income). Along the process of structural change, urban area grows faster than urban population, leading to a fall in the average urban density (plots (d) and (e) of Figure 6). This is the outcome of two different forces. On one hand, this is the natural consequence of rural productivity growth solving the ‘food problem’: higher rural productivity frees up farmland for cities to expand, lowering farmland rents relative to income. Moreover, as workers spend less on rural goods, they can afford larger homes and spend relatively more on housing. The city expands outwards at a fast rate. As land at the city fringe is getting cheaper (relative to income), the city expands by adding a less and less dense suburban fringe over time, contributing to the fall in average urban density (plot (e) of Figure 6).

On the other hand, rising urban productivity leads to a reallocation of workers away from the dense center towards the fringe—contributing further to the fall in average urban density. With a rising urban income, workers move towards the suburbs to enjoy larger homes despite a rising opportunity cost of commuting time. This is so because they optimally choose faster commuting modes when moving towards the suburbs. Thus, although the mechanisms are entirely different, both rural and urban productivity growth contribute to urban sprawl and falling urban density in this experiment.

Regarding land rents, the reallocation of workers away from agriculture and the fall in the relative price of rural goods exerts downward pressure of the price of farmland. Thus, land rents are reallocated away from the rural part towards the urban part (plot (f) of Figure 6)).

To sum up, beyond the well-known predictions regarding employment shares across sectors, our theory is able to qualitatively reproduce the salient facts described in Section 2 for France regarding

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23 In our formulation of the commuting costs, $a$ is a transformation of the different commuting costs parameters but one can always set the commuting efficiency $c_r$ to target a given $a$ (see Equation 7).

24 In Miles and Sefton (2020), the average speed of commuting in England has been multiplied by almost 5 since 1840—in line with our baseline experiment described below. Parisian data detailed in Appendix B.9 shows a similar increase.
the expansion of the urban area, the evolution of urban density and land values.

Figure 6: Baseline numerical illustration.

Notes: Simulation with 1.25% constant productivity growth in both sectors, $c > 0$, and $s = 0$.

Rural versus urban productivity growth. To disentangle further the mechanisms at play, it is useful to investigate the model’s implications when only rural or urban productivity growth occurs. Figure 7 shows selected model’s outcomes with only rural productivity growth—$\theta_r$ growing at 1.25% per year, while $\theta_u$ is set to unity throughout. The qualitative implications are similar to the baseline illustration. Workers move away from the rural sector, the rural good and farmland are getting less expensive and urban density falls despite rising urban population. However, the city sprawls less: without urban productivity growth, there is less reallocation away from central locations towards the fringe. Average urban density mostly falls due to the addition of lower density habitat at the urban fringe where land gets cheaper—central density falling significantly less.

Figure 8 shows model’s outcomes with only urban productivity growth—$\theta_u$ growing at 1.25% per year, while $\theta_r$ is set to unity throughout (resp. a high value for comparison). Here, the qualitative implications are more widely different from the baseline illustration. Urban productivity growth leads to urban expansion in area but not in population: many rural workers are required to satisfy subsistence needs and feed the population (plot (a) of Figure 8). The city expands in area

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25 Urban population might even very slightly fall as more workers are required to produced subsistence needs with
as higher urban productivity reallocates urban workers away from the center towards the urban fringe. As the demand for land at the fringe rises, so does the price: farmland is getting more expensive. This, in turn, increases suburban density, mitigating the overall fall in urban density. As a consequence, central density is falling more than the average one (plot (b) of Figure 8). With only urban productivity growth, rural land rents (as a share of income) do not fall and there is no reallocation of land values towards the urban areas (plot (c) of Figure 8). Thus, rising urban productivity and faster urban commutes are not sufficient to account for the evolution of urban densities and land rents across space.

Lastly, it is important to note that the reallocation of urban residents away from the center towards the suburbs is significantly stronger at a higher level of rural productivity. In other words, the interaction between rural and urban productivity matters for the area expansion of cities (plot (a) of Figure 8). If rural productivity is low, people spend most of their resources on rural necessity goods, limiting their ability to expand their housing space when urban productivity increases. As a consequence, rising urban productivity reallocate significantly less people towards the suburbs and cities stay dense despite higher urban wages. To the opposite, when rural productivity is high enough, rising urban productivity expands the urban area much more as urban residents afford larger housing space. In this sense, beyond the direct effect of rural productivity on urban expansion, rural productivity is also crucial as it provides the necessary incentive for people to relocate towards the city fringe and use faster commutes when urban productivity increases.

To sum up, our numerical illustrations show how, in the presence of subsistence needs, agricultural productivity growth not only matters for urbanization and the reallocation of workers away from the rural sector, but it is also essential to replicate the large historical decline in urban density, the fall in farmland prices (relative to income) and the reallocation of land rents towards urban areas.

**Labor push versus labor pull.** In the baseline illustration, the driver of structural change is rural productivity growth combined with subsistence needs for rural goods—a model where rising productivity frees up resources for the urban sector to expand (‘rural labor push’). An alternative view on structural change would emphasize a rising demand for (luxury) urban goods as income rises (‘urban labor pull’). In our set-up, this would correspond to a high $s$ relative to $c$. For comparison, we simulate the economy with a value for $s$ twice as big as $c$ ($s = 2c = 1.2$), such that, keeping all other parameters to their baseline values, the initial share of employment in the rural sector remains close to 60%. Under such preferences, Figure 9 shows the model dynamics following rising productivity in both sectors. While such a calibration can generate employment shares broadly in line with the evidence, it cannot generate the observed fall in urban density. As income increases, the spending share on housing falls as the income elasticity of housing demand is low: workers are willing to reduce their housing size to consume more of the urban good. Thus, the city does not expand much in area to host more numerous urban workers and urban density does not fall. Urban density tends to increase due to the reallocation of workers towards the urban center (plot (b) of less land available for agriculture.
Figure 7: Numerical illustration with only rural growth.

Notes: Simulation with 1.25% constant rural productivity growth and constant urban productivity; \( c > 0 \) and \( \bar{s} = 0 \).

The line with circles corresponds to the simulation with rural productivity being equal to the last period value, while the others correspond to the simulation with rural productivity being equal to the initial period value.

Figure 8: Numerical illustration with only urban growth.

Notes: Simulation with 1.25% constant urban productivity growth and constant rural productivity; \( c > 0 \) and \( \bar{s} = 0 \).

Figure 9: Numerical illustration with \( \bar{s} > c > 0 \).

Notes: Simulation with 1.25% constant productivity growth in both sectors. The preference parameters are such that \( \bar{s} = 2c > 0 \), while keeping the initial rural employment share close to 60%.
Figure 9): as they shrink their housing size, urban workers relocate away from the suburbs towards central locations, increasing central density—the opposite of the data.\textsuperscript{26} A high enough subsistence need is thus important for urban density to decline as it leads to an increase in the housing spending share following structural change. Note also that the evolution of the spending share on housing is informative regarding the relative magnitude of $c$ and $s$ (plot (b) of Figure 6 and plot (c) of Figure 9). An increasing share of housing spending, as in the data (see the calibration description in Section 4), points towards a calibration where $c$ is significantly larger than $s$.

4 Quantitative Model

We develop a quantitative version of the model to account for the process of structural change and urban expansion in France. The quantitative model is calibrated in 10-year steps using French historical data since 1840. Data are described in detail in Appendix B. We implement first a single city economy without agglomeration/congestion forces. One can interpret the following quantitative simulations as aggregate outcomes for the ‘average’ French city.

We present in this section results from an extended model which is better suited to match the data. We describe the extensions in detail in Appendix C. First, we consider a surface instead of a line segment as given land endowment. The city is circular with endogenous radius $\phi$ and area $\pi \phi^2$. Second, we allow for a more flexible parametrization of the commuting costs and of the construction costs faced by land developers. The latter are such that the housing supply elasticities depend on the location within the city (as in Baum-Snow and Han (2019)). Lastly, we consider a dynamic version of the model where households maximize their lifetime utility with borrowing and lending in a risk-free asset, which is in net-zero supply. Given a discount factor $\beta$, this pins down the path of the equilibrium real interest rate in our simulations and allows the computation of land values beyond rents. The relevant model for the current section is thus the one defined in Appendix C.1.4.

4.1 Calibration

The technology parameters are calibrated externally and the remaining parameters are set to match some outcomes observed in the data. While some parameters are jointly determined to minimize the distance between the model’s outcomes and a set of specified moments in the data, we provide, for sake of space, the main intuitions behind the identification of the model’s parameters. Details of the joint estimation of parameters $\{\nu, \gamma, c, s, a\}$ and the minimization procedure are provided in Appendix C.2.4. The parameter values for the baseline simulation of the quantitative model are summarized in Table 1.

\textsuperscript{26}Suburban (fringe) density does fall in this experiment (plot (b) of Figure 9). The same mechanisms as in our baseline illustration also play a role: farmland is getting cheaper at the city fringe due to structural change.
Table 1: Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>Total Space</td>
<td>1.0</td>
</tr>
<tr>
<td>$L_0$</td>
<td>Total Population in 1840</td>
<td>1.0</td>
</tr>
<tr>
<td>$\theta_0$</td>
<td>Initial Productivity in 1840</td>
<td>1.0</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Labor Weight in Rural Production</td>
<td>0.75</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Land-Labor Elasticity of Substitution</td>
<td>1.0</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Preference Weight for Rural Consumption Good</td>
<td>0.03</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Utility Weight of Housing</td>
<td>0.3</td>
</tr>
<tr>
<td>$c$</td>
<td>Rural Consumption Good Subsistence Level</td>
<td>0.74</td>
</tr>
<tr>
<td>$g$</td>
<td>Initial Urban Good Endowment</td>
<td>0.21</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount Factor</td>
<td>0.94</td>
</tr>
<tr>
<td>$\xi_l$</td>
<td>Elasticity of commuting cost wrt location</td>
<td>0.55</td>
</tr>
<tr>
<td>$\xi_w$</td>
<td>Elasticity of commuting cost wrt urban wage</td>
<td>0.75</td>
</tr>
<tr>
<td>$a$</td>
<td>Commuting Costs Base Parameter</td>
<td>2.25</td>
</tr>
<tr>
<td>$\epsilon_r$</td>
<td>Housing Supply Elasticity in rural area</td>
<td>5.0</td>
</tr>
<tr>
<td>$\epsilon(0)$</td>
<td>Housing Supply Elasticity at city center</td>
<td>2.0</td>
</tr>
</tbody>
</table>

**Technology.** The share of land used in agriculture is set to 25%, $\alpha = 0.75$ as in Boppart et al. (2019). Boppart et al. (2019) provide an estimate very close to unity for the elasticity of substitution between land and labor in agriculture. Thus, as in our baseline model, rural production in the quantitative model is Cobb-Douglas but we perform sensitivity with respect to this elasticity of substitution in Appendix D.3.

The path for productivity in both sectors, $\theta_r$ and $\theta_u$, is calibrated to match its data counterpart using French sectoral data on production, employment and agricultural land use since 1840. The estimated path for $\theta_r$ and $\theta_u$ (displayed in Figure V in Appendix B.4) is in line with the evolution of the standards of living in France over the period. Such a path for productivity is consistent with the conventional view that the nineteenth century is characterized by faster productivity growth in non-agricultural sectors, manufacturing in particular, while agricultural productivity grew significantly faster post-World War II. More specifically, starting the agricultural crisis in the late nineteenth century, technological progress in the French agriculture was particularly slow and delayed relative to other countries, before catching up at a fast rate post World War II (Bairoch (1989)). Appendix C.2.1 details the smoothing of the $\theta_r, \theta_u$ data for use in the model.

**Demographics.** Population, $L_t$, is normalized to unity in the first period and set at each date to match the increase of the French population since 1840 according to Census data. Over the period considered, the French population roughly doubled and the increase in the labor force is of the same magnitude. Going forward, we use the projections for the French population by INSEE until 2040 and set a constant growth rate of 0.4% thereafter.

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271840 is the first date of observation for agricultural land use necessary to compute the path of rural productivity. Due to the normalization of price indices, $\theta_r$ and $\theta_u$ are set equal to unity in this initial period. The yearly path of $\theta$s in the data is smoothed to remove business cycles fluctuations.
Preferences. Given technology, demographics, and the commuting costs elasticities, the preference parameters \( \{ \nu, \gamma, c, s \} \) are jointly set such that the agricultural employment share and the housing spending share are in line with the data. More precisely, the subsistence needs in agriculture, \( c \), determines the initial agricultural employment share in 1840, while the preferences parameter towards the rural good, \( \nu \), determines the long-run employment share in agriculture. Similarly, the endowment of urban good, \( s \), determines the housing spending share for the year 1900 (23.7\% with a 5-year average around 1900)—our initial period of observation regarding consumption expenditures, while the preference parameter towards housing services, \( \gamma \), determines the housing spending share in recent years (31.4\% in 2015).

The last preference parameter, the discount factor \( \beta \), is irrelevant for the equilibrium allocation but pins down the rate of interest and thus matters for the value of land at each date. It is set externally to 0.94 such that the value of agricultural land over income matches the data in 1840.

Housing supply conditions. Existing estimates of the housing supply elasticities, \( \epsilon \), typically vary between 2 and 5, depending on the location as well as on the estimation technique (see, among others, Albouy et al. (2018), Combes et al. (2017) and Baum-Snow and Han (2019)).\(^{28}\) Baum-Snow and Han (2019) provides evidence of the within-city variation of the housing supply elasticities, ranging from about 2.5 at the CBD to about 5 at the fringe of cities. For the purpose of the quantitative analysis, we extend the baseline theory by allowing location-specific housing supply elasticities, \( \epsilon(\ell) \) with \( \partial \epsilon(\ell) / \partial \ell \geq 0 \) (see Appendix C.1.3 for details). This is meant to capture that it might be more costly for developers to build closer to the center than in the suburbs or the rural part of the economy. Following Baum-Snow and Han (2019), we set an elasticity of 2 at the CBD and 5 at the fringe and the rural area, and we report sensitivity to this value in Appendix D.1.\(^ {29} \) For comparison purposes, we also perform sensitivity analysis with a constant elasticity of housing supply, \( \epsilon = 3 \), with results displayed in Appendix D.2.

Commuting costs. For the purpose of our quantitative analysis, we expand the commuting choice model by introducing a more general spending cost on commuting \( f \), which still depends on the mode choice \( m \) but also on the commuting distance \( \ell \) and the labor costs \( w_u \) (see Appendix C.1.2 for details). Intuitively, beyond its speed, the pecuniary cost of a commuting mode depends on the distance traveled (e.g. cost of gasoline/energy) as well as the overall level of wages (e.g. wage of the bus driver). Under some parametric assumptions, commuting costs under an optimal mode choice are of the following form (comparable to Equation (7)),

\[
\tau(\ell) = a \cdot w_u^{\xi_u} \cdot \ell^{\xi_\ell},
\]

where the elasticities of commuting costs to income, \( \xi_u \), and to distance, \( \xi_\ell \), are both positive and

\(^{28}\)With Cobb-Douglas production of housing using land and structure, there is a mapping between the elasticity \( \epsilon \) and the land share in housing production. Typical estimates of the land share varies between 0.2 and 0.3, corresponding to elasticities between 2 and 4.

\(^{29}\)We assume that the elasticities \( \epsilon(\ell) \) evolve linearly from the central value to the fringe value. Results are barely affected when performing sensitivity analysis with respect to the functional form of \( \epsilon(\ell) \).
below unity. The parameter $a$ is inversely related to the efficiency of the commuting technology. We use individual level commuting data detailed in Appendix B.9 to calibrate the elasticities $\xi_w$ and $\xi_\ell$. In the model, the elasticity of speed to commuting distance is equal to $1 - \xi_\ell$. We find in Appendix B.9.1 that this elasticity is precisely estimated within a narrow range around 0.45—depending on the sample used and the controls.\(^{30}\) Thus, $\xi_\ell$ is set externally to 0.55.

The elasticity of commuting costs to income $\xi_w$ is tied to the evolution of urban speed when average income increases. More precisely, $(1 - \xi_w)$ is the elasticity of speed to wage income at a given commuting distance. Using the individual commuting data detailed in Appendix B.9.2, one can estimate the percentage change in speed over 30 years for a given commuting distance. Over the period 1984-2013, this increases is 11% for an increase in measured urban productivity of 44%—yielding an estimate for $\xi_w = 1 - \frac{11}{44}$. Thus, $\xi_w$ is set externally to 0.75.

The remaining parameter $a$ is estimated to make the urban area, $\pi \phi^2$, represent 18% of rural land in the recent period—the measured artificial land is 18% of the land used for agriculture in 2015. Results are not very sensitive to $a$ as long as $\pi \phi^2$ is a relatively small fraction of the available land.

4.2 Results

We present model’s predictions over the period 1840-2020 under the baseline calibration with only one representative city. Data counterparts, when available, are described in Appendix B.

Structural change. Figure 10 shows that our model is able to account for the patterns of structural change observed in the data. Rising rural productivity reallocates labor away from the rural sector and makes rural necessity goods less valuable. The relative price of rural goods falls as productivity increases. Our model fits the data on the historical evolution of the relative price remarkably well, despite not being targeted (Figure 10b). Moreover, rising income leads to a reallocation of spending away from rural goods towards the urban good and housing services: the spending share on the rural good gradually falls, the share spent on the urban good continuously increases, and so does the spending share on housing services, although at a slower speed (Figure 10c). Overall, the spending share patterns are broadly in line with the data if one abstracts from fluctuations in the interwar period (see Figure VI in Appendix B.5).

Urban expansion. Figure 11 shows the model’s outcomes regarding the evolution of city size (area versus population) and the average urban density. For comparison with data on urban expansion, the plots start in 1870—normalizing the 1870-value to unity. In line with the data, cities expand much faster in area than in population (Figure 11a). While our model does not account for the full observed expansion of the urban area, it explains a very large fraction. As a consequence, the model predicts a large fall in average urban density—density is divided by more than 6 since 1870, slightly less than in the data (Figure 11b). As structural change slows down, so does the fall in urban density.

\(^{30}\) Commuting data also shows that the relationship between speed and commuting distance is very close to log-linear as in the model. See same Appendix.
Figure 10: Structural change.

Notes: Outcomes of the baseline simulation of the quantitative model where parameters are set to the values of Table 1. Corresponding data for the employment share, the relative price of rural goods and spending shares are described in appendices B.2, B.3 and B.5. The relative price is normalized to 1 in 1950.

Figure 11: Urban expansion.

Notes: Outcomes of the baseline simulation of the quantitative model where parameters are set to the values of Table 1. Plots start in 1870 for comparison with data. Corresponding data for urban population, area and average density are described in Appendix B.7. Data and model outcomes are normalized to 1 in 1870 and shown on a log-scale.

Density across space. Figure 12 shows the model’s predictions for density in different locations. Figure 12a depicts the evolution of the central density and the density at the fringe of the city (relative to the average), where densities are normalized to 1 in 1840 for readability. The fall in average density is driven both by a fall in central density and a fall in density at the urban fringe. Central density falls because households find it worth to use faster commuting modes and to move towards the suburbs as their income rises. The fall in density at the suburban fringe is the natural consequence of structural change: the reallocation of workers away from agriculture combined with less valuable agricultural goods puts downward pressure on the price of farmland. Households can

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31 The fringe of the city center is at 15% of the radius of the city in 1840.
afford larger homes in the suburban parts of the city. The latter mechanism, more specific to our theory, is crucial to generate a fall in average density that is larger than the fall in the central one—in line with the Parisian data discussed in Section 2. Our model predicts that the overall fall in the central density is about 60% of the fall in the average density—in the ballpark of the estimates for Paris. Lastly, one can measure the density gradient by distance deciles within the urban area, both in the data and in the model in the recent period. The model’s predictions are shown in Figure 12b. The shape of the curve is very close to an exponential (fitted curve) as in the data, and the value of the coefficient of the fitting curve is 0.16 as in the data (see Appendix B.7.4). Thus, our quantitative model provides a reasonable fit of the data regarding the density of urban settlements across time and space.

![Figure 12: Density across space.](image)

**Notes:** Outcomes of the baseline simulation of the quantitative model where parameters are set to the values of Table 1. Density in different urban locations (left plot) is normalized to 1 in 1840 for readability. Density of the city center is computed on a circle ending at 15% of the initial city radius in 1840. The density gradient (right plot) shows model-implied urban density at different deciles of distance from the center in 2015, with density in the last decile normalized to unity. The fitted-curve is the exponential curve that fits model outcomes the best for data comparison.

**Commuting speed and the ‘agricultural productivity gap’**. Our model with endogenous commuting costs generates predictions regarding the evolution of commuting speed across time. Moreover, the marginal urban worker, which has the longest commute, needs to be compensated relative to the rural worker. Our model thus predicts an endogenous urban-rural wage gap, which depends on the city fringe ($\phi$) and the endogenous commuting costs in this furthest away location. These predictions are shown in Figure 13. Over time, our model generates almost a five-fold rise in the average commuting speed (Figure 13a). We collected historical data on the use of different commuting modes for Paris to provide an estimate of the evolution of the average commuting speed in the Parisian urban area (see Appendix B.9 for details). The overall increase in average speed since 1840 predicted by the model is of a similar magnitude than in the Parisian data.\(^{32}\)

\(^{32}\)Miles and Sefton (2020) find a very similar increase in speed for the U.K.. Such historical data are not available...
the overall increase, the predictions over the whole period line up relatively well with the evolution of commuting speed in the Parisian area. The increase by a factor of about 2 until 1930 reflects the more intensive usage of public transport and their increase in speed over this period (from the initial horse-drawn omnibus to the metro). The later increase, more specifically post-World War II, reflects the increasing car usage. Overall, the model provides predictions regarding the evolution of the average urban speed that are of reasonable magnitude.

Following Gollin et al. (2014), Figure 13b shows the ‘agricultural productivity gap’—a monotonic transformation of commuting costs at the fringe of the city proportional to the urban-rural wage gap, $w_u/w_r$. We compute the raw ‘agricultural productivity gap’ as,

$$\text{Raw-APG} = \frac{L_r/L_u}{VA_r/VA_u},$$

where $VA_i$ denotes the value added in sector $i$. The value predicted by the model for the recent period, around 1.5, is in line with the values computed by Gollin et al. (2014) for France—lying in between their Raw-APG and Adjusted-APG. Computing the Raw-APG for the entire sample period directly from historical national accounts data, we find that our model falls short of the entire gap, especially for the initial years, but explains a large fraction since 1960.\(^{33}\) Our quantitative model suggests that spatial frictions combined with location-specific housing can generate urban-rural wage gaps of a significant economic magnitude. It also provides insights on the persistence of fairly large gaps even in developed countries, where labor misallocation is arguably less relevant.

**Land values and housing prices.** Figure 14 shows the model’s predictions for land values and housing prices. Figure 14a shows the reallocation of land value across rural and urban use. Due to structural change, the value of rural land relative to urban land fell dramatically. In the model, while the value of agricultural land constituted more than 80% of the total land value, it is less than 10% nowadays. This is broadly in line with data from Piketty and Zucman (2014) even though our model partly misses the timing of the reallocation around the time of World War II—arguably due to war destructions.\(^{34}\) Importantly, the value of urban land (per unit of land) increased faster in the recent decades. This mirrors the evolution of the housing price index since 1870 (Figure 14b), whose shape reminds of the hockey-stick shown in Figure 5b. The model generates about half of the increase in housing prices described in Knoll et al. (2017) post-World War II.

### 4.3 Sensitivity analysis

In order to shed further light on the mechanisms at play and discuss the sensitivity of our results to the different elements of the model, we perform alternative experiments. More specifically, these

\(^{33}\) Using wage data, Sicic (1992) provides estimates of the urban-rural wage gap in France over the period 1852-1911. Like in the U.K., he finds a significant increase of the gap over the period, in line with our predictions.

\(^{34}\) To compute the urban land value in the data, we multiply the housing wealth by the share of land in housing, whose average is 0.32 in the data for the period 1979-2019. More details are provided in Appendix B.6.
experiments aim at showing how structural change and the use of faster commutes interact in driving
the urban expansion. We also discuss the robustness of our findings to the production side in the
rural and housing sector.

Figure 13: Commuting speed and the ‘agricultural productivity gap’.
Notes: Outcomes of the baseline simulation of the quantitative model where parameters are set to the values of Table 1. The average urban commuting speed (left plot) is the density-weighted average of speeds across urban locations (see Appendix C.1.6 for definition, normalization to 1 in 1840). Estimates for Paris are detailed in Appendix B.9.3. The agricultural productivity gap (right plot) is defined as \( \frac{L_r/L_u}{VA_r/VA_u} \).

Figure 14: Land values and housing price.
Notes: Outcomes of the baseline simulation of the quantitative model where parameters are set to the values of Table 1. Land and housing values are computed as the discounted sum of future land rents in each location. Corresponding data (dashed) are based on Piketty and Zucman (2014) and described in more detail in Appendix B.6. The real housing price index averages the purchasing housing prices across locations (deflated using a model implied GDP-deflator). Details on the computation are provided in Appendix C.1.5.

34
The role of rural productivity growth. To emphasize further the crucial role of technological progress in agriculture and structural change for our results, it is useful perform sensitivity analysis with a lower rural productivity growth. We perform simulations with a stagnating (resp. slowly growing) rural productivity, where the growth rate of $\theta_r$ is 2% (resp. 20%) of the baseline at each date. Results of these simulations are shown in Figure 15 for some variables of interest together with the baseline simulation for comparison. With low improvements of the rural technology, the urban density falls significantly less and might even increase if rural productivity stays sufficiently low (Figure 15a). The growth of population and urban productivity puts pressure on land in the rural area to feed an increasing and richer population. This increases the relative price of rural goods and the price of farmland at the urban fringe (Figure 15c)—preventing the city to expand.\textsuperscript{35} Furthermore, facing higher price of rural goods, households reduce their housing spending share to feed themselves, reducing the demand for urban land. These forces tend to make the city much denser than our baseline—more so at the urban fringe due to rising farmland values (Figure 15b). Thus, urban density might increase despite the reallocation of urban workers towards the less dense part of the city as they commute faster due to rising urban productivity. It is worth emphasizing that population growth, by putting pressure on land, makes improvements in agricultural productivity even more crucial to generate a sizable expansion in urban area.

This simulation does not say that improvements in commuting technologies do not matter for the expansion in area of cities. However, it makes clear that they matter only when combined with rural productivity growth and structural change. The next experiment provides further insights on the quantitative role of commuting costs for our results.

The elasticity of commuting costs to income. To shed further light on the quantitative im-

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\textsuperscript{35}In the simulation with stagnating rural productivity, the city even shrinks in size. Workers move away from cities despite urban productivity growth as more rural workers are needed to feed the increasing population.
Importance of falling commuting costs and rising commuting speed, we set the elasticity of commuting costs to income, $\xi_w$, to unity, $\tau(\ell) = a.w_u.\ell^{\xi_u}$. With such a calibration, the fraction of wages devoted to commuting in given location does not fall with rising urban productivity, contrary to our baseline. This is so because the speed of commuting does not increase with a rising opportunity cost of time (urban wage).\(^{36}\) When compared to our baseline, this illustrates the quantitative role of the use of faster commutes when urban productivity increases. Figure 16 shows the results in this alternative calibration together with our baseline for comparison purposes. Figure 16a makes clear that increasing the elasticity of commuting costs to income limits the increase in the average commuting speed over the period.\(^{37}\) As the cost of faster commutes increases more than in our baseline, urban workers do not relocate away from central locations towards the suburban part of the city. This severely limits the expansion in area of the city relative to the baseline and the average urban density falls significantly less (Figure 16b).

![Figure 16](image_url)

**Figure 16: Sensitivity to the elasticity of commuting costs to income.**

*Notes:* The elasticity of commuting cost to income, $\xi_w$, is set to 1. All other parameters are kept to their baseline value of Table 1. Simulation for the baseline calibration shown in dotted for comparison.

Thus, when combined with rural productivity growth, the use of faster commutes and the corresponding decline in commuting costs (as a share of the urban wage) is quantitatively important to account for the overall decline in urban density—particularly so in central locations. In this alternative simulation, as the urban area expands much less but urban population grows essentially as much due to structural change, urban land values and housing prices increase much more than in our baseline (Figure 16c). This mirrors the role of improvements in commuting modes to limit the increase in urban land values emphasized in Heblich et al. (2018) and Miles and Sefton (2020).\(^{38}\)

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\(^{36}\)In this knife-edge calibration, workers do not switch to faster modes at a given location when facing an increase in their wage: the increase in the operating cost of faster commutes offsets the benefits due to a rising opportunity cost of time.

\(^{37}\)The average speed still increases slightly as, due to structural change, workers locate in suburban locations where they are willing to use faster commuting modes.

\(^{38}\)Higher urban housing prices generate an agricultural productivity gap about twice as large as in our baseline simulation in the recent period. Equivalently, the urban resident at the fringe faces much higher commuting costs.
The elasticity of substitution between land and labor in the rural sector. Our baseline simulation assumes a unitary elasticity of substitution between land and labor, $\sigma = 1$. Values used in the literature typically range between 0 and 1 (Bustos et al. (2016) and Leukhina and Turnovsky (2016)). We perform sensitivity analysis with a lower value of 0.25. We also show results for a high value of 4 to enlighten further the quantitative importance of the adjustment of land values at the fringe of the city for our results.\(^{39}\) Results are displayed in Appendix D.3 for variables of interest (Figure XXXIII). With a lower elasticity of substitution, the rental price of farmland falls more (increases less) following structural change as land and labor are more complement in the rural sector. As the opportunity cost of expanding the city is lower, the urban area increases more and the average urban density falls more. This is driven by a larger fall of density in the cheaper suburban parts. With $\sigma = 0.25$, the model matches the expansion in area and the corresponding decline in average density observed in French cities since 1870. To the opposite, if land and labor are more substitutes ($\sigma = 4$), the reallocation of workers away from agriculture puts less downward pressure on the value of farmland, limiting the expansion of the urban area and the decline in density, which falls short of the data. These experiments further illustrate the importance of the farmland price adjustment at the urban fringe to understand the reallocation of land use.

The housing supply elasticity. Our baseline simulation features location-specific housing supply elasticities with a lower elasticity at the city center relative to the fringe. As sensitivity analysis, we set the elasticity to 3 in all locations, in the mid-range of empirical estimates.\(^{40}\) Results regarding the time evolution of the aggregate variables of interest—employment, relative price of rural goods, urban area, average urban density and land values—are barely affected. The most noticeable difference is that a constant housing supply elasticity generates a city center much denser relative to the suburban part. Compared to our baseline simulation, a more elastic housing supply at the center leads to a larger provision of housing in these locations. As a consequence, the gradient of population density is significantly steeper than in the data (Figure XXXII in Appendix D.2).

4.4 Extensions

Agglomeration and congestion. We introduce urban agglomeration forces by assuming that the urban productivity increases externally with urban employment, $\theta_u(L_u) = \theta_u \cdot L_u^\lambda$. We set $\lambda = 0.05$, in the range of empirical estimates for France (Combes et al. (2010)). Other parameters are left identical to the baseline calibration for comparison, adjusting the initial value of $\theta_u$ to have the same initial urban productivity. For variables of interest, results in presence of agglomeration forces are displayed in Figure 17 together with the baseline simulation. While the city expands slightly more in area, there is barely no effect of agglomeration forces on urban population. The faster increase in the urban wage due to agglomeration forces increases urban housing demand and reduces urban commuting costs (as a share of income). This relocates urban households towards

\(^{39}\)A higher $\sigma$ limits the fall of farmland values at the fringe of cities when workers move towards the urban sector.

\(^{40}\)This corresponds to a land share in housing of 25%, slightly lower than the average in the data over the period 1979-2019.
the suburbs where they can enjoy larger homes and the city sprawls more. However, a higher urban income makes also rural goods more valuable increasing rural workers’ wage almost one for one. General equilibrium forces thus prevent workers’ reallocation towards cities. They also make rural land more valuable—mitigating the area expansion of the city. As a consequence, despite higher incomes driven by urban expansion, the economy with agglomeration forces behave quantitatively similarly to our baseline.

Figure 17: Agglomeration and congestion forces.

Notes: The solid line represents outcomes in presence of agglomeration forces, with parameter $\lambda = 0.05$. The solid line with dots represents outcomes in presence of congestion forces, with parameter $\mu = 0.05$. Other parameters set to their baseline value of Table 1 up to a normalization of the initial urban productivity. For comparison, outcomes of the baseline simulation are shown with a dotted line.

We also consider additional urban congestion costs by assuming that commuting costs are increasing with urban population, $a(L_u) = a \cdot L_u^\mu$. This summarizes the potential channels through which larger cities might involve longer and slower commutes for a given commuting distance. We set $\mu = 0.05$ and we re-scale the constant $a$ to have the same initial value for the commuting costs, leaving other parameters to their baseline values. The evolution of the variables of interest is shown on the same Figure 17 for comparison. Congestion forces move the equilibrium in the opposite direction of agglomeration forces. They reduce the expansion in area and the extent of suburbanization. By increasing commuting costs, they also increase urban housing prices. However, via general
equilibrium forces, they also make rural goods and rural land less valuable—severely mitigating the direct effect of congestion costs on urban expansion.

**Commuting distance and residential location.** Guided by the structure of French cities, our baseline results hinge on the assumption of a monocentric model where urban individuals commute to the city center to work. While endogeneizing firms location across space is beyond the scope of the paper, one can still partly relax the monocentric assumption by assuming that commuting distance, $d(\ell)$, does not map one for one with residential distance $\ell$ from the central location. Using data available for the recent period to investigate the link between commuting distance and residential location (see Appendix B.9.2 for details), we find that households residing further away do commute longer distances on average. However, commuting distance increases less than one for one with the distance of residence from the city center. Data also show that individuals residing very close to the center commute longer distances than the distance of their home from the central location.\footnote{Data also show that commuting distance increases less with the distance of residence from the center in larger cities—pointing towards a larger dispersion of employment away from the center in larger cities. See Appendix B.9.2.}

Based on these observations, we model commuting distance $d(\ell)$ in a reduced-form way as follows,

$$
d(\ell) = d_0(\phi) + d_1(\phi) \cdot \ell, \quad (29)
$$

with $d_0(\phi)$ being a positive and increasing function of $\phi$ satisfying $\lim_{\phi \to 0} d_0(\phi) = 0$, and $d_1(\phi)$ being a decreasing function belonging to $(0, 1)$ with $\lim_{\phi \to 0} d_1(\phi) = 1$. $d_0$ represents the (minimum) commuting distance traveled by an individual living in the center, while $d_1$ is the slope between commuting distance and residential distance from the center. This specification fits recent data well. It also makes sure that at the limit of $\phi \to 0$, the city is monocentric as all the jobs must be centrally located. It is important to note that commuting costs are now defined as,$^2$

$$
\tau(\ell) = a \cdot w_u^{\xi w} \cdot (d(\ell))^\xi. 
$$

As detailed in Appendix D.5, we make the following parametric assumptions guided by the data: $d_0(\phi) = d_0 \cdot \phi$, with $d_0$ calibrated to 5% and $d_1(\phi) = \frac{1}{1 + d_1 \cdot \phi}$, with $d_1$ calibrated to 2. For the sake of space, details and results of this extension are relegated to Appendix D.5. We find that our results are not much affected (Figure XXXVI). Quantitatively, the city expands more in area in the last decades under this specification of the commuting distance, bringing the model closer to the data. As a consequence of this larger sprawling, the average urban density falls more. This is driven by a larger fall of central density, the most noticeable difference relative to our baseline monocentric model. With urban expansion, residents in central locations end up commuting larger distances—implicitly due to the reallocation of jobs away from the center—, this makes central locations less attractive relative to suburban ones.

**Multiple cities.** We extend our quantitative model with a single region/city to allow for $K$ different
regions. The spatial structure in each region $k$ is identical to the one-city version.\textsuperscript{43} Regions are heterogeneous only in their urban productivity—with $\theta_{u,k}$ the urban productivity in region/city $k$. Workers are freely mobile within and across regions and labor markets clear globally. Urban and rural goods are freely traded within and across regions and goods markets clear globally. For quantitative evaluation, we consider 20 regions, corresponding to the 20 largest French cities in 1870. Each region is endowed with the same land area as our baseline and aggregate population preserves the baseline endowment of land per head.\textsuperscript{44} City-specific urban productivities, $\theta_{u,k}$, are set to match the distribution of population of the different cities over the period 1870-2015, while keeping aggregate urban productivity equal to its baseline value estimated from French historical national accounts. Other parameters are set to their baseline values of Table 1. Details of this extension together with the numerical solution method are provided in Appendix D.6.

For aggregate variables of interest (aggregate sectoral employment, aggregate land use, relative price of rural goods/farmland, ...), the model with multiple regions behaves quantitatively very similarly to the baseline with only one city. Intuitively, the one-city model describes well the dynamics of aggregate variables for a representative ‘average’ city. Thus, we focus on the dispersion of city size and density, which are novel outcomes relative to our baseline. Detailed outcomes of the model are relegated to Appendix D.6. Beyond the targeted distribution of population across cities, the model does a fairly good job at reproducing the distribution of urban area and average urban density across time and space. Figure 18 plots the log of average urban density in a given city against its data counterpart for the dates where it is observed in the data (1870, 1950, 1975, 1990 and 2015).\textsuperscript{45} The model predicts that, over time, for a given city, urban density falls as urban population increases—in line with the predictions of our one-city model. In the cross-section, more populated cities are however denser as they feature higher housing prices. At a given date, the reallocation of workers towards a more productive city does not imply the general equilibrium effects on rural (good and farmland) relative prices at the heart of the time series evolution of urban density when cities grow in size. Importantly, both predictions, over time and in the cross-section, are qualitatively in line with the data discussed in Section 2. Quantitatively, the model does notably better in the time-series than in the cross-section. At a given date, more populated cities are significantly denser in the model than they are in the data (relative to less populated ones).

As last sensitivity analysis, we consider the model with multiple-cities where, in each city, residential location does not map one for one into commuting distance. Using the same parametric assumptions, we use the reduced-form Equation (29) for each city $k$ of fringe $\phi_k$. Results and details are relegated to Appendix D.6. The fit between model and data improves slightly under this specification for

\textsuperscript{43}Each region $k$ is made of urban and rural land, with only one (potential) city per region. For each region, the city center is centrally located within each region and regions are assumed large enough in area such that cities do not expand in neighboring regions.

\textsuperscript{44}This makes sure that, with homogeneous urban productivity, the version with multiple regions behaves like the one-city model in each region.

\textsuperscript{45}Figure XXXVII in Appendix D.6 does the same for urban population (targeted) and area (non-targeted). The model-implied outcomes are defined up to a constant of normalization defining the unit of measurement. In Figure 18 they are normalized such that the average across all observations matches the data counterpart.
commuting distance. In line with the data presented in Appendix B.9, commuting distances in the
center (resp. at the fringe) are larger (resp. lower) in larger cities in this specification relative to the
monocentric model. This, in turn, increases the area of more populated cities in the cross-section
at a given date, reducing their average density and bringing the model closer to the data. More
populated cities in the model are still noticeably denser than in the data but less so compared to
the monocentric model.

5 Conclusion

This paper develops a spatial general equilibrium model of structural change with endogenous land
use and studies its implications for urbanization. We document a persistent fall of urban density in
French cities since 1870 and show that the theoretical and quantitative predictions of the model are
broadly consistent with the data. The quantitative version of our theory calibrated to French data
explains (at least) three fourths of the urban area expansion and of the decline in average urban
density, about half of the rise in housing prices, and most of the land value reallocation from rural
to urban since the mid-nineteenth century. Novel predictions regarding urban density across space
line up relatively well with available data. Predicted agricultural productivity gaps in recent times
are also of reasonable magnitude.

Agricultural productivity growth is shown to be crucial for the results, since it reduces the price of
land at the urban fringe and frees up resources to be spent on housing. As a consequence, while
workers reallocate away from agriculture, cities grow faster in area than in population and land
prices do not rise very rapidly. Faster commuting modes also play an important and complementary
role but only when combined with rural growth and structural change. When rural productivity is high, they allow households to live further away from their workplace and enjoy larger homes, contributing significantly to the decline in urban density, particularly at the city center.

Our baseline framework assumes a monocentric urban structure where all workers commute from their residential location to the city center. While French cities exhibit the qualitative features of monocentric cities, such an urban structure certainly remains an approximation. In particular, data shows that commuting distance increases with residential distance to the center but less than one for one. This suggests that workers sort into jobs and residential locations that are closer from each other. We believe that relaxing the monocentric structure remains an important step to better account for the expansion of cities and the evolution of density across urban locations. However, we leave for future research a theory that jointly determines firms and workers location decisions across the urban space within our framework.

We also believe that our approach can be used to study the aggregate implications of policies regulating land use and urban planning. Such policies are also likely to play a role in explaining the evolution of housing prices in recent years, which our current setup cannot fully replicate. The general equilibrium structure of our quantitative spatial model makes it well suited to conduct such policy counterfactuals.
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