Microeconomics IV

Part III. Imperfect Information

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PART III. IMPERFECT INFORMATION

- First Welfare Theorem: all competitive equilibrium allocations are Pareto efficient
(invisible hand mechanism).
- Assumptions behind it:
  - markets are competitive,
  - no externalities,
  - perfect information.
- Market failures:
  - consumption/production externalities,
  - public goods,
  - asymmetric information.

Part III. Imperfect information

1. Choice under uncertainty
   1.1 State-contingent budget constraints
   1.2 Preferences under uncertainty
   1.3 Choice under uncertainty
   1.4 Fair insurance
   1.5 Risk diversification
2. Asymetric information: adverse selection
3. Asymetric information: moral hazard

TOPIC 7. CHOICE UNDER UNCERTAINTY

- So far, optimization problems considered had no uncertainty.
- However, in the real world people often make decisions with uncertainty about future prices, future wealth, or other agents' decisions.
- In this section, we study the choice problem under uncertainty using a two-state version of our consumer's choice model.
- Optimal responses: insurance purchase, risk diversification.
- Example:
  - 2 possible states of nature: car accident (loss of L€), no car accident.
  - Probabilities for each state: π_a, π_{na}.
  - Insurance: get K€ if accident by paying γK€ as insurance premium.

State-contingent budget constraints

Definitions

A contract is ***state contingent*** if it is implemented only when a particular state of Nature occurs.
A ***state-contingent consumption plan*** specifies the consumption to be implemented when each state of Nature occurs.

- Example:
  - Consumption if no accident: $c_{na} = M - \gamma K$
  - Consumption if accident:
    $c_a = M - L - \gamma K + K \Rightarrow K = \frac{c_a - M + L}{1-\gamma}$

$$\Rightarrow c_{na} = M - \gamma \left(\frac{c_a - M + L}{1 - \gamma} \right)$$

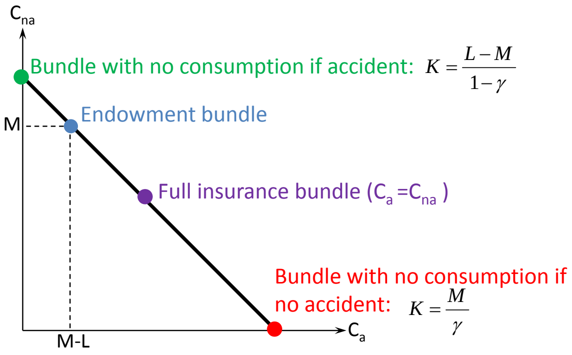
$$\Leftrightarrow c_{na} = \frac{M - \gamma L}{1 - \gamma} - \frac{\gamma}{1 - \gamma} c_a$$

State-contingent budget constraints

Car Insurance example

- State-contingent budget constraint in car insurance example:

$$c_{na} = \underbrace{\frac{m - \gamma L}{1 - \gamma}}_{\text{intercept}} - \underbrace{\frac{\gamma}{1 - \gamma}}_{\text{slope}} c_a$$



Preferences under Uncertainty

- To know what is the agents' choice, we need to know their preferences about the different state-contingent consumption plans.
- Utility across state-contingent consumption plans is a function of the consumption levels and probabilities at each state, $U(c_1, c_2, \pi_1, \pi_2)$.

Definition

A utility function $U(c_1, c_2, \pi_1, \pi_2)$ satisfies the ***expected utility*** or ***von Neumann–Morgenstern*** property if it can be written as the weighted sum of the utility at each state, where the weights are the probabilities of each state:

$$U(c_1, c_2, \pi_1, \pi_2) = \pi_1 v(c_1) + \pi_2 v(c_2)$$

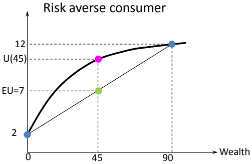
It satisfies the independence property, which means that the utility in a given state is independent of the utility in other states.

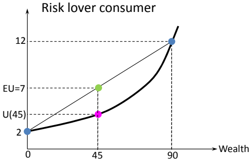
Risk aversion

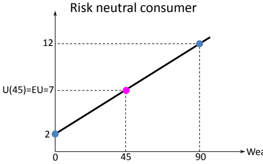
Definition

We say an agent is ***risk averse*** if the expected utility of wealth is lower than the utility of expected wealth, ***risk lover*** if it is higher, and ***risk neutral*** if it is equal.

- Example:
  - Lottery: 90€ with probability 1/2, 0€ with prob 1/2.
  - Utility levels: U($90) = 12, U($0) = 2.
  - Expected utility: EU=1/2*12+1/2*2=7.
  - Expected money value: EM=1/2*90+1/2*0=45.





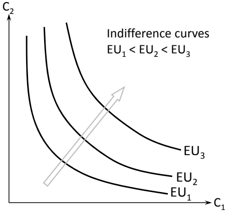


Indifference Curves

- State-contingent consumption plans that give equal expected utility are equally preferred and on the same indifference curve.
- Slope of indifference curves:

$$EU = \pi_1 U(c_1) + \pi_2 U(c_2) \Rightarrow dEU = \pi_1 \frac{\partial U(c_1)}{\partial c_1} dc_1 + \pi_1 \frac{\partial U(c_2)}{\partial c_2} dc_2$$

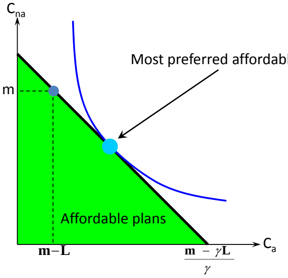
$$\pi_1 \frac{\partial U(c_1)}{\partial c_1} dc_1 + \pi_2 \frac{\partial U(c_2)}{\partial c_2} dc_2 = 0 \Rightarrow \frac{dc_2}{dc_1} = -\frac{\pi_1 \partial U(c_1)/\partial c_1}{\pi_2 \partial U(c_2)/\partial c_2}$$



Choice under uncertainty

- The optimal choice under uncertainty is the most preferred affordable state-contingent consumption plan.
- In the car insurance example, the optimal consumption plan is where the slope of indifference curves is tangent to the budget constraint:

$$\frac{\pi_a \partial U(c_a)/\partial c_a}{\pi_{na} \partial U(c_{na})/\partial c_{na}} = \frac{\gamma}{1-\gamma}$$



Fair insurance

Definition

We say an insurance is ***fair or competitive*** if the expected economic profit of the insurer is zero, or, equivalently, if the price of a 1€ insurance is the probability of the insured state.

- Car insurance example:

$$\underbrace{\gamma K}_{\text{revenues}} \quad \underbrace{-\pi_a K - (1-\pi_a)0}_{\text{expected expenditures}} \quad = 0 \Rightarrow \gamma = \pi_a$$

- If the insurance is fair, the optimal choice of risk-averse consumers is full insurance:

$$\frac{\pi_a}{\pi_{na}} \frac{\partial U(c_a)/\partial c_a}{\partial U(c_{na})/\partial c_{na}} = \frac{\pi_a}{1-\pi_a} \Rightarrow \frac{\partial U(c_a)}{\partial c_a} = \frac{\partial U(c_{na})}{\partial c_{na}}$$

Hence, for risk averse consumers, $c_a = c_{na}$.

Unfair insurance

Unfair Insurance

Definition

We say an insurance is ***unfair*** if the insurer makes positive expected economic profits.

- If the insurance is unfair, the optimal choice of risk-averse consumers is less than full insurance:

$$\underbrace{\gamma K}_{\text{revenues}} \quad \underbrace{-\pi_a K - (1-\pi_a)0}_{\text{expected expenditures}} > 0 \Rightarrow \frac{\gamma}{1-\gamma} > \frac{\pi_a}{\pi_{na}}$$

Hence, $\frac{\pi_a}{\pi_{na}} \frac{\partial U(c_a)/\partial c_a}{\partial U(c_{na})/\partial c_{na}} = \frac{\gamma}{1-\gamma}$ implies that

$$\frac{\partial U(c_a)}{\partial c_a} > \frac{\partial U(c_{na})}{\partial c_{na}}$$

so, for risk averse consumers, $c_a < c_{na}$.

Diversification

- Asset diversification typically lowers (or keeps) expected earnings in exchange for lowered risk. This is going to be the case as long as the asset prices are not perfectly correlated across states.
- Example: two firms, two states (prob. 1/2), agent with 100€ to spend in firms' share.
  - Firm A: shares' cost 10€, profits per share in state 1 100€, in state 2 20€.
  - Firm B: shares' cost 10€, profits per share in state 1 20€, in state 2 100€.

	10 shares of A	10 shares of B	5 of A, 5 of B
Profits in 1	1000€	200€	600€
Profits in 2	200€	1000€	600€
Expected profits	600€	600€	600€

Part III Contents. Imperfect information

ASYMETRIC INFORMATION

- In the purely competitive markets, agents are assumed to have perfect information about all the exchange aspects.
- In some markets, however, this is clearly not realistic (medical services, used cars, insurance...):
  - A doctor knows more about medicine than the patient.
  - A used car seller has more information about the car than the potential buyer.
  - The buyer of an insurance knows much more about his/her risks than the insurer.
- We say that a market suffers from ***imperfect information*** if one of the sides does not have all the information about the exchange.
- We say that a market suffers from ***asymetric information*** if one of the sides has more information than the other.

Asymetric information inefficiency

- Under asymetric information, markets typically have less transactions than under the perfect information equilibrium; hence, we say that the equilibrium under asymetric information is inefficient.
- In this context, government intervention may be Pareto improving but it may also be Pareto worsening.
- The question is, then, whether government has more information than market participants and whether the costs associated to it are not too large.

Asymetric information applications

- ***Adverse selection*** refers to a situation where one of the sides cannot observe all the characterstics of a good (also known as ***hidden quality problem***).
- ***Signaling*** refers to a situation where the high/quality agent takes actions to different him/herself from the rest of agents.
- ***Moral hazard*** refers to a situation where one of the sides cannot observe the actions of the other side (also known as ***hidden action problem***).
- ***Incentives contracting*** consists on designing a system of incentives to prevent agents from taking undesired actions after the contract is signed.

TOPIC 8. ADVERSE SELECTION

- Consider a used-cars market with two types of cars, "lemons" and "peaches":
  - The reservation price of "lemons" sellers is 1000, while buyers are willing to pay 1200.
  - The reservation price of "peaches" sellers is 2000, while buyers are willing to pay 2400.
- Therefore, when sellers have perfect information all the cars get sold and the total surplus is positive:
  - "Lemons" are sold for an amount between 1000 and 1200.
  - "Peaches" are sold for an amount between 2000 and 2400.
- But what happens when sellers do not know the car type?

Akerlof model

- If buyers cannot distinguish "lemons" and "peaches" (but they know the proportions), how much are they willing to pay?
  - Let q denote the fraction of high-quality cars and $1-q$ the low-quality ones.
  - Therefore, the expected value for the buyer is $V^e = 1200(1-q) + 2400q$.
- When q is such that $V^e < 2000$,
  - The high-quality sellers leave the market.
  - Only the low-quality sellers stay in the market.
  - Knowing this, buyers are only willing to pay 1200.
- Hence, too many low-quality cars are expelled from the market, which reduces the exchange surplus. In other words, the presence of low-quality sellers imposes an external cost to high-quality sellers and buyers.

Akerlof model (2)

- How many low-quality cars are compatible with high-quality cars remaining in the market?
  - Buyers are willing to pay 2000 if q is such that $V^e = 1200(1-q) + 2400q \geq 2000$.
  - Hence, if $q < \frac{1}{3}$, only low-quality cars are sold.
- Pooling and seprating equilibrium:
  - In a **separating equilibrium** only of type of cars is traded or each type is traded at a different price.
  - In a **pooling equilibrium** both types of cars are sold at the same price.

Adverse selection with a continuum of types

- If the car quality x is uniformly distributed between 1000 and 2000 and buyers are willing to pay $x+300$ for a car of quality x.
- Which cars are traded in this case?
  - The expected quality is 1500 and the expected value for sellers is $1500 + 300 = 1800$.
  - Hence, sellers of quality above 1800 leave the market.
  - As a result, the expected value for the cars remaining in the market becomes $\$1400 + \$300 = \$1700$.
  - As a result, the sellers with quality between 1700 and 1800 abandon the market..
  - ...

Adverse selection with a continuum of types (2)

- Which cars will remain in the market?
  - Denote v_H the quality (or seller's value) of the best car in the carel valor.
  - The expected quality of the cars in the market is, thus, $V^e = 1000 + \frac{v_H - 1000}{2} = \frac{1000}{2} + \frac{v_H}{2}$.
  - Hence, the expected value for the seller is $\frac{1000}{2} + \frac{v_H}{2} + 300$.
  - Since this will also be the highest quality in the market,

$$\frac{1000}{2} + \frac{v_H}{2} + 300 = v_H \Rightarrow v_H = 1600$$

- Therefore the adverse selection expels all the cars with quality above 1600.

Adverse selection with quality choice

- Suppose now that sellers can choose the quality of the product they sell.
- Example: two types of umbrellas, high quality and low quality (not differentiable for the consumer).
  - Buyers' valuation is 14 for the high quality and 8 for the low quality.
  - Production cost is 11 for the high quality and 10 for the low quality.
- Is there any equilibrium in this market?

Adverse selection with quality choice (2)

- Is it possible an equilibrium where only low-quality umbrellas are sold?
  - No, because sellers would have a benefit equal to -2.
- Is it possible an equlibrium where only the high-quality umbrellas are produced?
  - If there are only high-quality umbrellas, buyers are willing to pay 14 and sellers obtain a profit of 3.
  - But sellers can have a benefit equal to 4 by producing low quality umbrellas.
  - Hence, it is not possible to only have high-quality producers.

Adverse selection with quality choice (3)

- Is it possible to have an equilibrium where both types are produced?
  - Let q denote the fraction of sellers producing high quality, where $0 < q < 1$.
  - Then, the buyers' expected value is $V^e = 14q + 8(1-q) = 8 + 6q$.
  - Since the high quality producers must have positive profits, $8 + 6q > 11q + 10(1-q) \Rightarrow q > 2/5$.
  - But note that sellers can always increase their profits by producing only low quality.
  - And when $q = 0$ buyers are only willing to pay 8.
  - There, an equilibrium with both types does not exist either!
- In this situation, the adverse selection problem completely destroys the market!

Signaling

- In a context with adverse selection, high-quality sellers have an incentive to signal their quality:
  - Methods: reference letters, warranties, advertisement.
- Example: labor market with high ability type and low ability type.
  - The marginal product of the high ability type is a_H and the low ability type is a_L, where $a_L < a_H$.
  - The fraction of high ability workers is h, while the fraction of low ability workers i $1 - h$.
- If firms can distinguish the two types (and workers are paid their marginal product)

$$w_H = a_H, \; w_L = a_L.$$

- But if firms cannot differentiate the two types, they will offer workers the expected marginal product:

$$w_P = (1 - h)a_L + ha_H.$$

- Since $w_P = (1 - h)a_L + ha_H < a_H$, high-quality workers may be willing to pay to send a credible signal.

Education as a signal

- Workers may want to use education as a signal:
  - Denote the education cost by c_H for high-education workers and c_L for low-education workers, with $c_L > c_H$.
  - Assume (just as a example!) that education does not change the productivity of workers.
- High-abilty agents want to get e_H units of education if it works as a signal:
  - the benefit is higher than the cost for high-ability workers: $w_H - w_L = a_H - a_L > c_H e_H$,
  - the benefit is higher than the cost low-ability workers: $w_H - w_L = a_H - a_L < c_L e_H$.
- In this situation, high-ability workers want to get education while low-ability workers do not.
- Therefore, education is useful to signal the type.
- Note that signaling solves the information asymetry problem but at a cost.

Warranties as a signal

- Example: used-cars market with high and low quality cars:
  - High-quality cars: sellers are willing to sell for 8 000, buyers are willing to pay 10 000.
  - Low-quality cars: sellers are willing to sell for 5 000, buyers are willing to pay 6 000.
- Suppose sellers have the possibility of offering a warranty, which costs 500/year for high-quality car owners and 2000/year for low-quality car owners..
  - Is there any warranty duration n such that only high-quality owners are interested in it?
  - High-quality owners are interested in offering it if

$$10000 - 500n \geq 6000$$

  - Low-quality owners are not interested in offering it if

$$10000 - 2000n < 6000$$

  - Hence, if $2 < n \leq 4$, a warranty allows low-quality sellers to differentiate themselves.

Advertising as a signal

- Consider now a market with asymetric information and high and low quality sellers.
- Suppose that consumers purchase the good only once if the quality is low and several times if quality is high.
- One possible way to signal the high-quality is to make launching sales or offer free samples.
- Another signaling option is to advertise the product; this is the case if there an advertising expenditure G such that:
  - G is lower than the extra benefits obtained by high-quality sellers due to the advertisement.
  - G is higher than the extra benefits obtained by low-quality sellers due to advertisement.

Part III Contents. Imperfect information

TOPIC 9. MORAL HAZARD

- ***Moral hazard*** is the agents' optimal response to a change in the risk loss and it is consequence of the asymetric information.
- For example, when someone buys a bike theft insurance, it he/she more likely to leave the bike unlocked?
  - In case of robbery, the cost is lower when insurance purchased, so the incentives to protect are lower.
- It is then a situation where
  - the lack of insurance is inefficient because there is an exogenous theft risk and consumers are risk averse,
  - but the insurance can also create other inefficiencies if agents modify their behavior.
-
- Insurers try to reduce the moral hazard problem:
  - Health insurance premium is higher for smokers;
  - Car insurance premium is lower for drivers with a good record of accidents.

The principal-agent model

- Consider a situation where
  - a worker (agent) is hired by an employer (principal) to perform a job;
  - there is a conflict of interests between the principal and the agent: the principal wants a high effort by the agent to maximize benefits, while the agent does not like effort;
  - effort is not observable.
- The objective is to analyze how the principal can give the agent the right incentives.
- Other principal-agent examples: lawyer and customer, auto repair shop and customer.

Incentives contracting

- The ***problem of the principal***: design the incentives contract to encourage an optimal effort by the worker.
  - Let e denote the effor of the agent and $y = f(e)$ the revenues of the principal.
  - An incnetives contract $s(y)$ specifies the worker compensation as a function of the principal revenues, y.
  - In this context, the principal benefits are

$$\pi_p = y - s(y) = f(e) - s(f(e)).$$

- At the same time, the principal has to take into account the ***agent participation constraint,*** so that the worker gets at least his/her reservation utility.
  - Denote by $\tilde{u}$ the worker's reservation utility.
  - Denote by $c(e)$ the cost of exerting the effort e.

Incentives contracting (2)

- The principal optimization problem is, then:

$$\max_{e} \pi_p = f(e) - s(f(e))$$

$$\text{s.t. } s(f(e)) - c(e) = \tilde{u} \text{ (participation constraint)}$$

- The optimal contract specifies an effort level e^* such that the marginal benefit of the principal is equal to the marginal cost of the worker:

$$f'(e^*) = c'(e^*)$$

- How to encourage to worker to choose $e = e^*$?
  - The contract $s(y)$ has to satisfy the ***incentive compatibility constraint***:

$$s(f(e^*)) - c(e^*) \geq s(f(e)) - c(e) \text{ for any } e \geq 0$$

Incentives contracting (3)

- Commercial rental contracts:
  - The principal asks for a fixed rent R and the worker keeps all the extra profits: $s(f(e)) = f(e) - R$.
  - Thus, the worker maximizes its revenues $s(f(e)) - c(e) = f(e) - R - c(e)$, and chooses the efficient effort level e^* such that $f'(e^*) = c'(e^*)$.
  - The principal will choose the highest possible R^* considering the agent participation constraint $f(e^*) - R^* - c(e^*) = \tilde{u}$

- Variable wage contracts (assuming that e is observable even if it is not verifiable):
  - Set the agent pay equal to $s(e) = we + K$.
  - If $w = f'(e^*)$, the worker chooses the optimal effort level e^*.
  - The principal then chooses$K$ such that the participation constraint is satisfied.

Efficiency wages

- To motivate workers to choose a high effort, a firm may be willing to pay wage above the equilibrium level.
- If workers do not perform well and they are fired, they will earn a lower wage in the next job.
- If all firms choose to pay efficiency wages, the mechanism still works: there is unemployment so, if fired, workers will not find a job immediately.
- Efficiency wages are likely to reduce the monitoring costs of firms as well as their hiring costs.
- In a context with adverse selection, they may also be useful to attract the most talented workers.

A model with efficiency wages

- The worker utility is $U = \sqrt{w} - g$, where $g = 2$ if effort is high, and $g = 1$ if effort is low.
- The probability that the worker is caught doing a low effort is q.
- If the wage offered by the firm is w^* and the equilibrium one is w_0,
  - the participation constraint is

$$\sqrt{w^*} - 2 \geq \sqrt{w_0} - 1 \Leftrightarrow \sqrt{w^*} \geq \sqrt{w_0} + 1,$$

  - and the incentive compatibility constraint is

$$\sqrt{w^*} - 2 \geq q(\sqrt{w_0} + 1) + (1-q)\left(\sqrt{w^*} - 1\right)$$

$\Leftrightarrow$

$$\sqrt{w^*} \geq \sqrt{w_0} + \frac{1}{q}.$$

Unemployment benefits in the efficiency wages model

- Suppose now that if the worker loses her job, there is a probability $1-t$ to become unemployed, with unemployment subsidy d.
- In this situation, the incentive compatibility constraint is

$$\sqrt{w^*}-2 \geq q\left(t\left(\sqrt{w_0}+1\right)+(1-t)\sqrt{d}\right)+(1-q)\left(\sqrt{w^*}-1\right)$$

$\Rightarrow$

$$\sqrt{w^*}=t\left(\sqrt{w_0}-\sqrt{d}-1\right)+\sqrt{d}+\frac{1}{q}+1$$

- As a result,
  - $t\downarrow \Rightarrow w^*\uparrow$.
  - $d\uparrow \Rightarrow w^*\uparrow$.

Other situations with insurance

- Insurance: when people get an insurance they are less likely to be cautious.
  - To reduce the moral hazard problem, insurance companies introduce a deductible F (paid by the agent in case of accident).
- Bank loans and investment projects:
  - Firms may choose a risky project once they get the loan because they don't have to pay back in case of failure.

Other incentives contracting examples

- Sales commissions: real state agents, insurance sellers...
- "Up or out" firm policy: members must leave the organization if they fail to achieve a certain rank within a certain period of time.
- Rankings of workers.

- Why don't we observe more incentives contracting examples:
  - Incentive schemes may not work as planned: to little quality if reward to production quantity.
  - *Free riding*. problems when production by teams instead of individuals.
  - Jobs with multiple tasks.
  - Participation of trade unions in design of job contracts.

THE END