Intermediate Microeconomics (22014)

Part III. General Equilibrium

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Outline Part III. General Equilibrium

1. Pure Exchange Economy (Varian, Ch 31)
    1.1 Edgeworth Box
    1.2 The Core
    1.3 Competitive Equilibrium
    1.4 Welfare Theorems
    1.5 Walras' Law
2. Production (Varian, Ch 32)
3. Welfare (Varian, Ch 33)

Topic 6. General Equilibrium

- Up until now, ***partial equilibrium*** analysis:
  - markets for goods analyzed in isolation, ignoring effect of other prices on the market equilibrium;
  - demand and supply functions of own price alone.
- In general, however, demand and supply in several markets interact to determine equilibrium prices of all goods.
  - Substitutes and complements.
  - People's income affected by goods sold.
- In top 6, ***general equilibrium*** analysis: all markets clear simultaneously.
- Considerations of Pareto efficiency and also of welfare distribution and "social preferences."

PURE EXCHANGE ECONOMY (Varian, Ch 31)

- Since very complex problem, simplifications adopted:
  - Only competitive markets studied, so that consumers and producers take prices as given.
  - Situations with, at most, two goods and two consumers.
  - First, ***pure exchange economy***: fixed endowments, no description of resources conversion to consumables.
  - Afterwards, ***production*** introduced into the model.
- Pure exchange economy:
  - Two consumers, A and B, two goods, 1 and 2.
  - Endowments of goods 1 and 2:

$$\omega^A = \left(\omega_1^A, \omega_2^A\right), \ \omega^B = \left(\omega_1^B, \omega_2^B\right).$$

  - Given a price vector (p_1, p_2), consumers choose their favorite affordable allocation (as in topic 1):

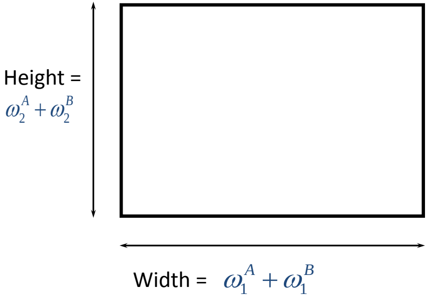
$$p_1 x_1^A + p_2 x_2^A \leq p_1 \omega_1^A + p_2 \omega_2^A$$
$$p_1 x_1^B + p_2 x_2^B \leq p_1 \omega_1^B + p_2 \omega_2^B$$

  - Prices must be such that allocations chosen are feasible:

$$x_1^A + x_1^B \leq \omega_1^A + \omega_1^B, \ x_2^A + x_2^B \leq \omega_2^A + \omega_2^B$$

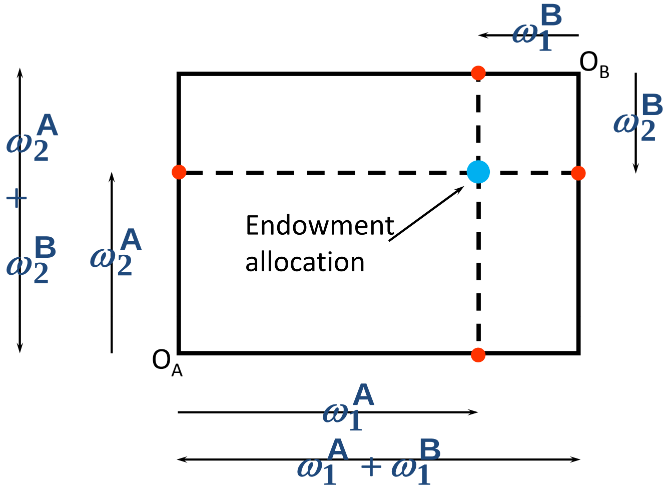
Edgeworth Box

- ***Edgeworth box*** is diagram showing all possible allocations of the available quantities of goods 1 and 2 between the two consumers.
  - The dimensions of the box are the quantities available of the goods.
  - The allocations depicted are the ***feasible allocations***.



Endowment allocation

- The ***endowment allocation*** is the before-trade allocation:

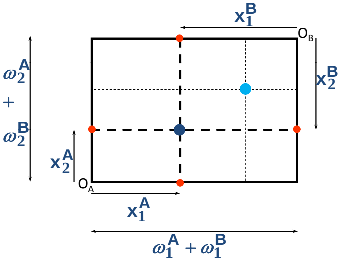


Feasible reallocations

- Which reallocation will consumers choose?
  - Feasible.
  - Pareto-improving over the endowment allocation.
- An allocation is ***feasible*** if and only if

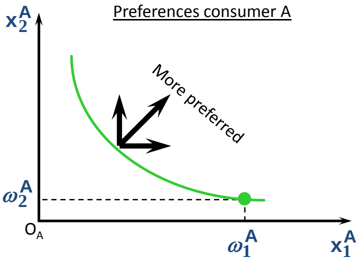
$$x_1^A + x_1^B \leq \omega_1^A + \omega_1^B$$
$$x_2^A + x_2^B \leq \omega_2^A + \omega_2^B$$

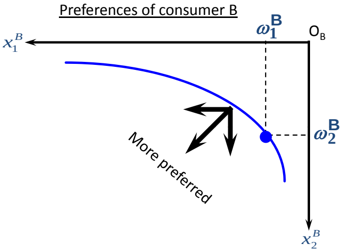
- All points in the box, including the boundary, represent feasible allocations of the combined endowments:



Pareto-improving allocations

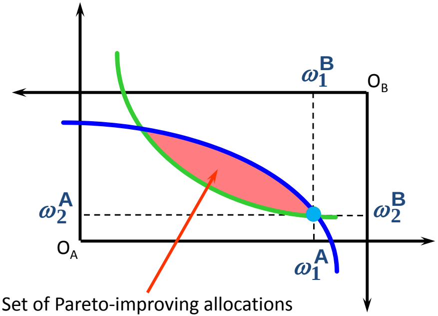
- An allocation is ***Pareto-improving*** over the endowment allocation if it improves the welfare of a consumer without reducing the welfare of another.
- Preferences of consumers A and B:





Pareto-improving allocations

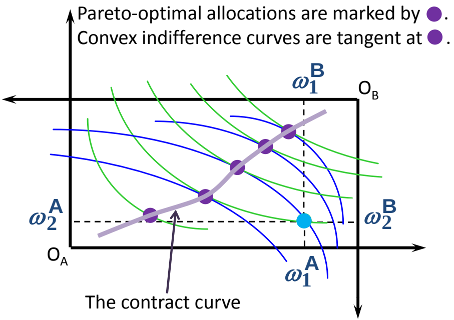
- An allocation is ***Pareto-improving*** over the endowment allocation if it improves the welfare of a consumer without reducing the welfare of another.



- Since each consumer can refuse to trade, the only possible outcomes from exchange are Pareto-improving allocations.

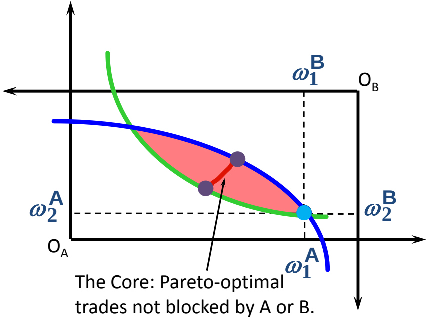
Contract curve

- An allocation is ***Pareto-optimal*** if the only way one consumer's welfare can be increased is to decrease the welfare of the other consumer.
- The set of all Pareto-optimal allocations is called ***contract curve***.



The Core

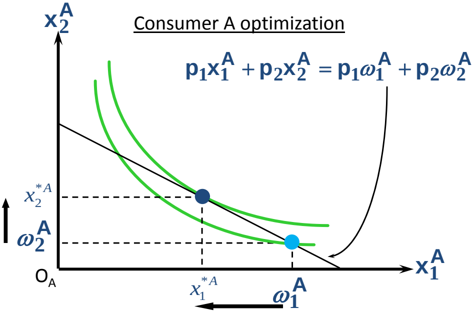
- ***The core*** is the set of all Pareto-optimal allocations that are welfare-improving for both consumers relative to their own endowments.



- Rational trade should achieve a *core* allocation.

Trade in competitive markets

- Specific core alloation achieved depends upon the manner in which trade is conducted.
- In perfectly competitive markets, each consumer is a price-taker trying to maximize her own utility given (p_1, p_2) and her own endowment:

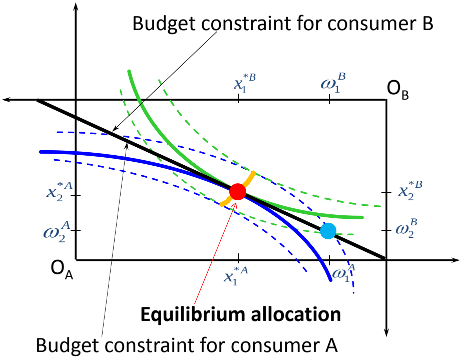


- Similarly for consumer B.

Trade in competitive markets

- At equilibrium prices p_1 and p_2, both consumers maximize their own utility and both markets clear:

$$x_1^A + x_1^B = \omega_1^A + \omega_1^B$$
$$x_2^A + x_2^B = \omega_2^A + \omega_2^B$$

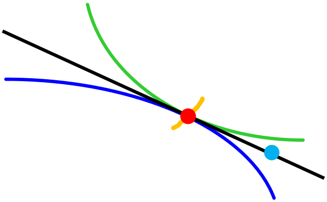


First fundamental theorem of welfare economics

Theorem

Given that consumers' preferences are well-behaved, trading in perfectly competitive markets implements a Pareto-optimal allocation of the economy's endowment.

- Note: Indifference curves are tangent, which implies that the equilibrium allocation is Pareto optimal.

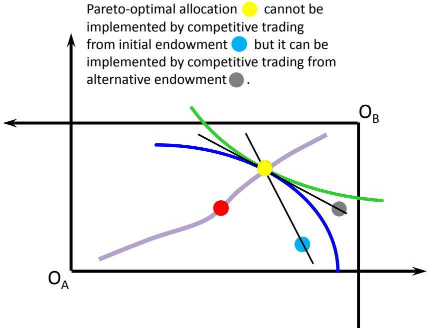


Second fundamental theorem of welfare economics

Theorem

Given that consumers' preferences are well-behaved, for any Pareto-optimal allocation, there are prices and an allocation of the total endowment that makes the Pareto-optimal allocation implementable by trading in competitive markets.

- In other words, any Pareto-optimal allocation can be achieved by trading in competitive markets provided that endowments are first appropriately rearranged.



Walras' Law

Theorem

*If consumer's preferences are "well-behaved", so that for any positive prices (p_1, p_2) consumers spend all their budget, the summed market value of excess demands is zero. This is **Walras' Law**.*

$$p_1 x_1^A + p_2 x_2^A = p_1 \omega_1^A + p_2 \omega_2^A$$
$$p_1 x_1^B + p_2 x_2^B = p_1 \omega_1^B + p_2 \omega_2^B$$

$\Rightarrow$

$$p_1 \left(x_1^A + x_1^B - \omega_1^A - \omega_1^B \right) + p_2 \left(x_2^A + x_2^B - \omega_2^A - \omega_2^B \right) = 0$$

Implications of Walras' Law

- One implication of Walras' Law for a two-commodity exchange economy is that if one market is in equilibrium then the other market must also be in equilibrium.

$$p_1\left(x_1^A + x_1^B - \omega_1^A - \omega_1^B\right) + p_2\left(x_2^A + x_2^B - \omega_2^A - \omega_2^B\right) = 0$$

$$\Rightarrow \text{If } x_1^A + x_1^B = \omega_1^A + \omega_1^B, \text{ then } x_2^A + x_2^B = \omega_2^A + \omega_2^B.$$

- Another implication of Walras' Law for a two-commodity exchange economy is that an excess supply in one market implies an excess demand in the other market.

$$p_1\left(x_1^A + x_1^B - \omega_1^A - \omega_1^B\right) + p_2\left(x_2^A + x_2^B - \omega_2^A - \omega_2^B\right) = 0$$

$$\Rightarrow \text{If } x_1^A + x_1^B < \omega_1^A + \omega_1^B, \text{ then } x_2^A + x_2^B > \omega_2^A + \omega_2^B.$$

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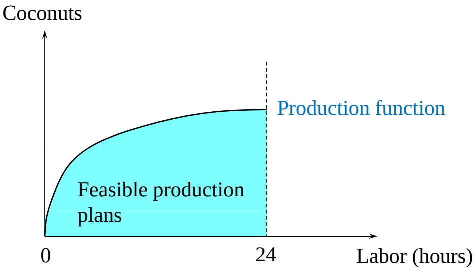
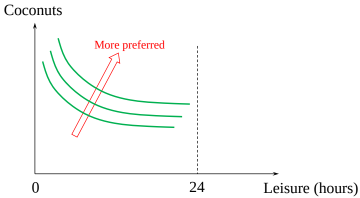
PRODUCTION (Varian, Ch 32)

- Add input and output markets, firms' technologies.
- Robinson Crusoe's Economy:
  - One agent: Robinson Crusoe.
  - Endowment: a fixed quantity of time.
  - Decision: use time for labor (production of coconuts) or leisure.
- Technology: coconuts are obtained from labor according to the production function $C = f(L)$.

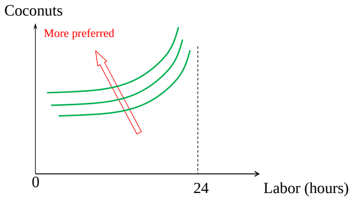


Robinson Crusoe's preferences

- Indifference curves in the leisure-coconut diagram: coconut is a good, leisure is a good:

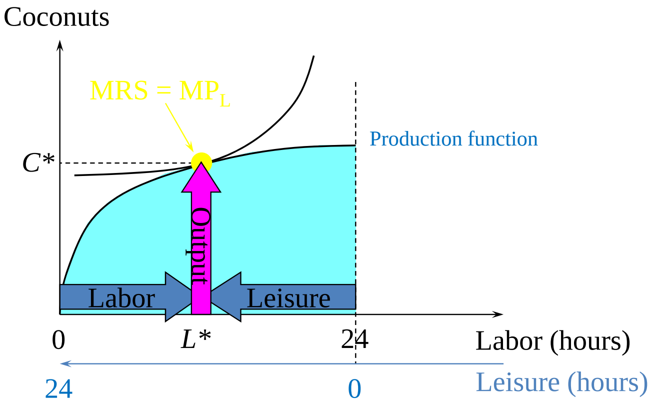


- Indifference curves in the labor-coconuts diagram: coconut is a good, labor is a bad.



Robinson Crusoe's choice

- Robinson chooses time allocation and, as a result, his consumption of coconuts:



Competitive equilibrium in the Robinson economy

- Robinson esquizofrenia:
  - We first consider Robinson as a ***profit-maximizing firm***, who takes prices as given and decides how much hours to hire and how much to produce.
  - Then, we consider Robinson as a ***utility-maximizing consumer*** who gets the firm profits and decides his hours of work and his consumption of coconuts.
- Let p be the coconuts price and w the wage rate.
  - Use coconuts as the numeraire good; i.e. price of a coconut $= 1$.

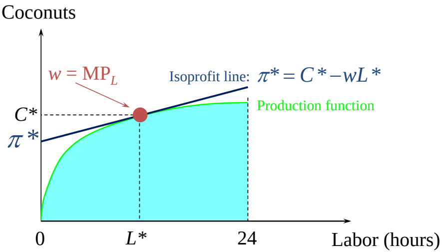
Robinson as a firm

- Optimization problem of the firm: given w, choose labor demand and coconut supply to maximize profits:

$$\max_{L} \pi = C - wL = f(L) - wL \Rightarrow MP(L^*) = w$$

  - Labor demanded: L^*, output supplied: $C^* = f(L^*)$.

- Graphically, firm demands L such that production function tangent to isoprofit line:



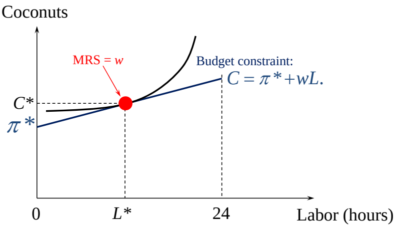
Robinson as consumer

- Optimization problem of the consumer: choose labor supply and coconut demand to maximize utility subject to the budget constraint:

$$\max_{C,L} U(C,L) \text{ s.t. } C = \pi^* + wL \Rightarrow \frac{\partial U(C,L)/\partial L}{\partial U(C,L)/\partial C} = w$$

  - Labor supplies: L^*, coconuts demanded: C^*.

- Graphically, consumer chooses C and L such that the indifference curve is tangent to the budget constraint:

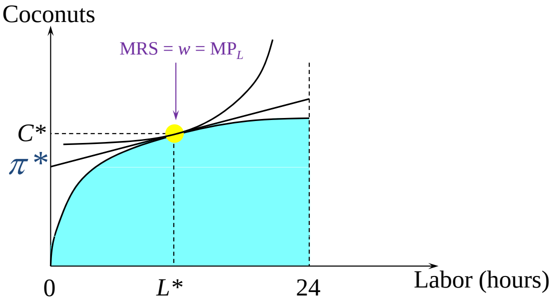


Market equilibrium

- In equilibrium, wage rate must be such that

quantity labor demanded = quantity labor supplied

(quantity output supplied = quantity output demanded)

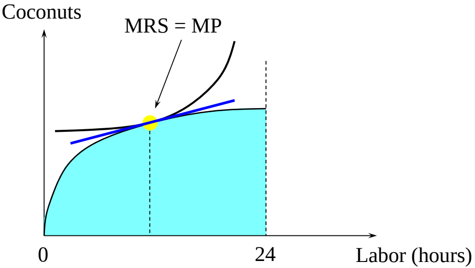


First Fundamental Theorem of Welfare Economics

Theorem

If consumers' preferences are convex and there are no externalities in consumption or production, a competitive market equilibrium is Pareto efficient.

- Pareto efficiency: $MRS = MP$:
  - Competitive equilibrium achieves Pareto efficiency: w is the common slope of the isprofit line and the budget constraint.



Second Fundamental Theorem of Welfare Economics

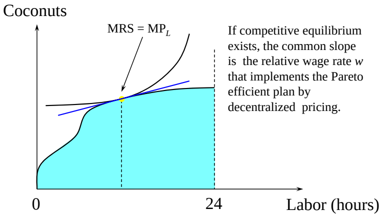
Theorem

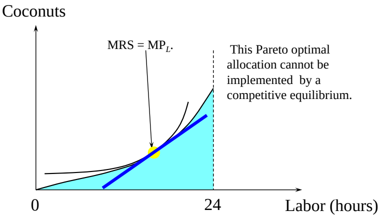
If consumers' preferences are convex, firms' technologies are convex, and there are no externalities in consumption or production any Pareto efficient economic state can be achieved as a competitive market equilibrium.

Non-convex technologies

- The First Welfare Theorems still holds if firms have non-convex technologies since it does not rely upon firms' technologies being convex.

- The Second Welfare Theorem does not hold if firms have non-convex technologies.





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WELFARE (Varian, Ch 33)

- Social choice: Different economic states will be preferred by different individuals. How can individual preferences be “aggregated” into a ***social preference*** over all possible economic states?

- Fairness: Some Pareto efficient allocations are “unfair” (for example, one consumer eats everything). Under what conditions, competitive markets guarantee that a ***fair*** allocation is achieved?

Social welfare functions

Let $u_i(x)$ be individual i's utility from overall allocation x.

- Utilitarian social welfare function:

$$W = \sum_{i=1}^{n} u_i(x)$$

- Weighted-sum social welfare function:

$$W = \sum_{i=1}^{n} a_i u_i(x), \ a_i > 0$$

- Minimax welfare function:

$$W = min\{u_1(x), u_2(x), ..., u_n(x)\}$$

Fair allocations

- An allocation is fair if it is
  - Pareto efficient
  - envy free (no agent prefers the allocation of other agents to their own).

- If every agent's endowment is identical, then trading in competitive markets results in a fair allocation (may not be true for non-competitive markets).